Class work 15. Algebra.

Algebra.

Let’s study the proposition
P = “I eat an apple and a banana for diner”
A = “I eat an apple”
B = ”I eat a banana”
P=A∧B
The true table is :

A	***B***	***A ∧ B***
T	T	T
T	F	F
F	T	F
F	F	F

The negation of this proposition can sound like
¬P = “It’s not true that I eat an apple and a banana for diner” and the true table for ¬P is (remember, ¬P is false if P is true and vice versa):

A	***B***	***¬P***
T	T	F
T	F	T
F	T	T
F	F	T

Let’s change A for ¬A and B for ¬B (again, if A was True, ¬A, will be False)

¬A	***¬B***	***¬P***
F	F	F
F	T	T
T	F	T
T	T	T

We can see that ¬P is false only when both ¬A and ¬B are false,
If

$$P = A \wedge B$$

the negation of P can be constructed as

$$\neg P = \neg A \vee \neg B$$

Which means I didn't eat apple for diner, or I didn't eat banana for diner, or I didn't eat anything. The negation of OR connective can be constructed in very similar way.

There is another logical connective, implication.
$P \to Q$. From P follows Q.
$P \to Q$ means "if P is true, Q is true as well." $P \to Q$ says **nothing** about what happens if P is false. The **only way** for $P \to Q$ to be false is if we know that P is true but Q is false. If P is false, anything can follow, so the proposition is true, but senseless.

$\boldsymbol{P}$	$\boldsymbol{Q}$	$\boldsymbol{P \to Q}$
T	F	F
T	T	T
F	T	T
F	F	T

P="number is divisible by 9", Q="number is divisible by 3"
We can create the proposition "is the number is divisible by 9, then it's divisible by 3".
Proof:
If number is divisible by 9, the number can be represented as $9k$ (where k is any natural number). But $9 = 3 \cdot 3$, so number can be rewritten as $3 \cdot 3 \cdot k$, and has a factor 3, so it's divisible by 3. Base on the proposition P, we proved, that Q is true, so $P \to Q$ is true.

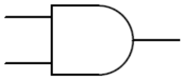
If P is false, $P \to Q$ doesn't mean anything. It's true, but it's not meaningful.
If P is true and Q is true, then the statement "if P is true, then Q is also true" is true.
If P is true and Q is false, then the statement "if P is true, Q is also true" is false.
Negation of implication.
Remember the true table of implication, let's see the negation

$\boldsymbol{P}$	$\boldsymbol{Q}$	$\boldsymbol{P \to Q}$	$\neg(\boldsymbol{P \to Q})$	$\boldsymbol{P}$	$\neg\boldsymbol{Q}$	$\boldsymbol{P}\wedge(\neg\boldsymbol{Q})$
T	F	F	T	T	T	T
T	T	T	F	T	F	F
F	T	T	F	F	F	F
F	F	T	F	F	T	F

$$\neg(\boldsymbol{P \to Q}) = \boldsymbol{P}\wedge(\neg\boldsymbol{Q})$$

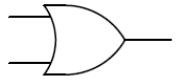
"It is not true that if the number is divisible by 9 it's also divisible by 3" is equivalent to "There is a number(s), that is divisible by 9 and not divisible by 3".
If $\boldsymbol{P \to Q}$ is true, it is not necessary that $\boldsymbol{Q \to P}$**.** If number is divisible by 3, it also can be not divisible by 9. But the proposition $\neg\boldsymbol{Q} \to \neg\boldsymbol{P}$ is true. If the nymber is not divisible by 3, it's definitely is not divisible by 9.

Logic gates:



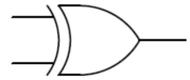
AND

A	B	Output
0	0	0
0	1	0
1	0	0
1	1	1



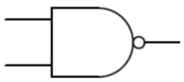
OR

A	B	Output
0	0	0
0	1	1
1	0	1
1	1	1



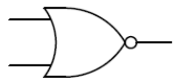
XOR

A	B	Output
0	0	0
0	1	1
1	0	1
1	1	0



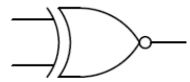
NAND

A	B	Output
0	0	1
0	1	1
1	0	1
1	1	0



NOR

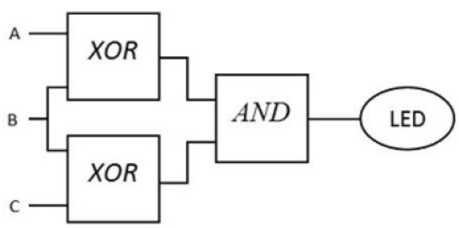
A	B	Output
0	0	1
0	1	0
1	0	0
1	1	0



XNOR

A	B	Output
0	0	1
0	1	0
1	0	0
1	1	1

1. The diagram below shows some circuit constructed of 3 logical chips (each with two inputs and one output; we draw them so that the inputs are on the left and the output, on the right). Can you determine for which values of inputs the LED will light up? [Hint: this is the same a writing a truth table for some formula....]



   Note: the wires connecting each of the chips and LED to the power source are not shown.

2. You meet two inhabitants: Peggy and Zippy. Peggy tells you that 'of Zippy and I, exactly one is a knight'. Zippy tells you that only a knave would say that Peggy is a knave.
   Can you determine who is a knight and who is a knave?

3. You meet two inhabitants: Marge and Zoey. Marge says, 'Zoey and I are both knights or both knaves.' Zoey claims, 'Marge and I are the same.'
   Can you determine who is a knight and who is a knave?

4. Check whether A ⇒ B and B ⇒ A are equivalent, by writing the truth table for each of them.

5. Check that A ⇒ B is equivalent to (NOT A) OR B (thus, “if you do not clean up your room, you will be punished” and “clean up your room, or you will be punished” are the same).

6. A teacher tell the student “If you do not take the final exam, you get an F”.
   Does it mean that
   (a) If the student does take the final exam, he will not get an F
   (b) If the student does not get an F, it means he must have taken the final exam.

7. Write the truth table for each of the following formulas. Are they equivalent (i.e., do they always give the same value)? (a) (A OR B) AND(A OR C) (b) A OR(B AND C).

8. You meet two inhabitants: Ted and Zeke. Ted claims, ‘Zeke could say that I am a knave.’ Zeke claims that it’s not the case that Ted is a knave.
   Can you determine who is a knight and who is a knave?

9. You meet two inhabitants: Ned and Zoey. Zed says that it’s false that Zoey is a knave. Zoey claims, ‘I and Ned are different.’
   Can you determine who is a knight and who is a knave?

10. You meet two inhabitants: Sue and Marge. Sue says that Marge is a knave. Marge claims, ‘Sue and I are not the same.’
    Can you determine who is a knight and who is a knave?

11. On the island of knights and knaves, you meet two inhabitants: Zoey and Mel. Zoey tells you that Mel is a knave. Mel says, “Neither Zoey nor I are knaves.” So who is a knight and who is a knave?

12. You meet two inhabitants: Betty and Peggy. Betty tells you that Peggy is a knave. Peggy tells you, ‘Betty and I are both knights.’
    Can you determine who is a knight and who is a knave?

13. You meet two inhabitants: Zed and Peggy. Zed says that Peggy is a knave. Peggy tells you, ‘Either Zed is a knight or I am a knight.’
    Can you determine who is a knight and who is a knave?

14. You meet two inhabitants: Zed and Alice. Zed tells you, ‘Alice could say that I am a knight.’ Alice claims, ‘It’s not the case that Zed is a knave.’
    Can you determine who is a knight and who is a knave?

15. While visiting the Knights and Knaves Island, you pass a beautiful garden where three islanders, Sam, Bob, and Tom are watching the sunset. You ask Sam, ”Are you a knight or a knave?” Sam is shy; you cannot hear his quiet answer. So you ask Bob, ”What did Sam say?” Bob answers, ”He said that he is a knave.” ”Don’t trust Bob! Bob is a knave!” screams Tom. Can you decide whether Bob and Tom are knights or knaves?

13. A traveler to the island of Knights and Knaves meets a group of five people (call them A, B,

C, D, E).
A says: “exactly one of us is a Knight”
B says: “exactly two of us are Knights”
C says: “exactly three of us are Knights”
D says: “exactly four of us are Knights”
E says: “all five of us are Knights”
Can you find out which of them are Knights?

14. You are in a maze on the island of knights and knaves. There are two doors: you know that one leads to freedom and one leads to certain doom. There are two guards nearby, and you happen to know that one is a knight and one is a knave, but you don’t know who is who. They allow you to ask one of them a single question before you choose a door — what do you ask?