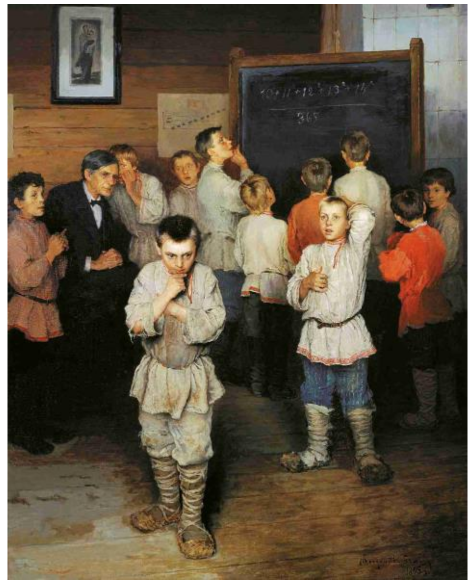
Class work 13. Algebra.

Algebra.



$$\frac{10^2+11^2+12^2+13^2+14^2}{365}=$$

Find GCD (GCF) and LCM for numbers

a. 222 and 345.
b. $2^2\cdot 3^3\cdot 5$ and $2\cdot 3^2\cdot 5^2$

Can we apply the same strategy to find CF and CM for algebraic expressions? (In this case the concept of GCD and LCM cannot be applied.) For example, can CF and CM be found for expressions $2x^2y^5$ and $4x^3y^2$? x and y are variables and can't be represented as a product of factors, but they itself are factors, and the expression can be represented as a product:

$2x^2y^5=2\cdot x\cdot x\cdot y\cdot y\cdot y\cdot y\cdot y,\qquad 4x^3y^3=2\cdot 2\cdot x\cdot x\cdot x\cdot y\cdot y\cdot$

$A=(Factors, 2x^2y^5)=\{2,x,x,y,y,y,y,y\}, B=(Factors, 4x^3y^3)=\{\,2,2,x,x,x,y,y\}$

Common devisors are any product of $A\cap B=\{\,2,x,x,y,y\}$.
What about common multiples? Product of all factors of both numbers (or the product of two numbers) will be the multiple, but minimal common multiple will be the product of the

$$A\cup B=\{2,2,x,x,x,y,y,y,y,y\}$$

$$\frac{2x^2y^5}{2\cdot x^2y^2}=y^3;\qquad \frac{4x^3y^2}{2\cdot x^2y^2}=2x;$$

$$\frac{4x^3y^5}{2\cdot x^2y^5}=2x;\qquad \frac{4x^3y^5}{4\cdot x^3y^3}=y^2;$$

Algebraic fraction

Algebraic fraction are fraction $\frac{A}{B}$ $(B\neq 0)$ whose numerator and denominator are algebraic expressions (not necessarily polynomials). For example:

$$\frac{3x^2+y}{y^2-5x+2};\qquad \frac{\frac{1}{x}-3}{y+\frac{1}{y}}$$

Several properties of these expressions:

$$\frac{A}{1} = A; \qquad \frac{A}{B} = \frac{A \cdot C}{B \cdot C} \quad (C \neq 0); \qquad -\frac{A}{B} = \frac{-A}{B} = \frac{B}{-A}$$

1. Add fractions:

   Example:

   $$\frac{2}{x^2a} + \frac{3}{a^2x} = \frac{2a}{a^2x^2} + \frac{3x}{a^2x^2} = \frac{2a+3x}{a^2x^2}$$

   $a.\ \frac{1}{a} + \frac{1}{b};$ $\qquad b.\ \frac{2}{x} - \frac{3}{y};$ $\qquad c.\ \frac{x}{a} + \frac{y}{b};$

   $d.\ \frac{5a}{7} - \frac{b}{x};$ $\qquad e.\ \frac{1}{2a} - \frac{1}{3};$ $\qquad f.\ \frac{1}{a} - \frac{1}{bc};$

2. Transform the following fraction, so that the sign before fraction is changed to the opposite:

   Example:

   $$-\frac{x}{x-3} = \frac{x}{-(x-3)} = \frac{x}{3-x}$$

   $a.\ \frac{1-x}{3};$ $\qquad b.\ -\frac{1}{2x+3y};$ $\qquad c.\ \frac{x-y}{x+y}$

   $d.\ \frac{-a-b}{x+y};$ $\qquad e.\ -\frac{a^2+1}{a-2};$ $\qquad f.\ -\frac{-x-y}{-a-b}$

3. Simplify fractions:

   1. $\frac{x-y}{y-x};$ 9. $\frac{2(a-b)}{3(b-a)};$ 17. $\frac{4mn(m-n)}{2m(n-m)};$ 24. $\frac{6a^2b^3(3-a)}{14ab^3(a-3)}.$

   2. $\frac{2x+2y}{4};$ 10. $\frac{3a+3b}{6a};$ 18. $\frac{4m-4n}{8mn};$

   3. $\frac{12ab}{6a-6b};$ 11. $\frac{2a-2b}{4a+4b};$ 19. $\frac{6x+6y}{3x-3y}.$

   4. $\frac{ax-bx}{cx+dx};$ 12. $\frac{ac+bc}{mc+nc};$ 20. $\frac{x^2}{x^2+xy};$

   5. $\frac{ab}{a-ab};$ 13. $\frac{m^2n}{m^2n-mn^2};$ 21. $\frac{ax-bx}{xy+x^2};$

   6. $\frac{p^2-p}{ap-bp};$ 14. $\frac{x^2-xy}{2xy+2x^2}.$

   7. $\frac{3xy}{3x^2a-3x};$ 15. $\frac{4m^2n}{6mn^2-8m^2n};$ 22. $\frac{3a^2+4ab}{9a^2b+12ab^2};$

   8. $\frac{4xy-x^2}{4x^2y-x^3y};$ 16. $\frac{2mn-6m^2}{12m^2n-4mn^2};$ 23. $\frac{16p^3q^3-24p^2q^4}{12p^2q^3-8p^3q^2}.$

Math. Logic

A speech of a person or a text written on paper contain sentences. This is the way how we exchange the information between us. The information in every sentence can be a true fact, false, or sometime we just can't say is it true or false. For example, the sentence:
"The Earth is rotating around the Sun" is true.
"Paris is the capital of Germany" is false.
"Math is fun!" or "What time is it?" are the sentences we can't tell either it's true or false. Can you tell which sentence is "true", "false", or we can't tell:

a. "22 is an even number"
b. "44 is an odd number"
c. "1001 is a cool number!"

Let's define "a statement" as a sentence about which we can tell (sometime after difficult process of proving) either it is true or false. For example, our first sentence "The Earth is rotating around the Sun" was proved to be true after hundreds of years of discussions. The second sentence, "Paris is the capital of Germany", can be proved wrong after we will check it in the dictionary (assuming that we never took geography class). As for the third example, how we can tell is 1001 a cool number? What is "cool"? for whom? Base on the definition, the sentence "22 is an even number" is a statement, and this is a true statement. "44 is an odd number" is also a statement, but the false one. "1001 is a cool number!" is not a statement at all.

1. Which of the following sentences are statements?
   a. When is the first day of school this year?
   b. The 4^{th} of July is Independence Day.
   c. How beautiful is it!
   d. Washington, DC is a capital of the United States.
   e. The sum of five and three.
   f. Three times five is twenty-six.

2. Which of the following statements are true, and which are false?
   a. There are 31 days in each January.
   b. There are 28 days in each February.
   c. Sunday is followed by Tuesday.
   d. There are 7 days in each week.
   e. There are 7 letters in the word "table"
   f. The sum of all single digit natural numbers is equal to 45.
   g. Every 3-digit natural number is grater then 100.
   h. There is a greatest 5-digit natural number.
   i. There is a greatest natural number.
   j. There is a smallest natural number.

Let’s take a look at the statement “New York City is the capital of the United States”. We, definitely, can say is it True or False. Of cause it’s not true, we all know that the capital of the US is Washington, DC. So, we can say “it is not true, that New York City is the capital of the US”, or, in a little more usual language, “New York City isn’t the capital of the US”. The last statement is a true statement.

“New York City is the capital of the United States” False

“New York City isn’t the capital of the US” (negation) True

If the statemen is True, its negated version has to be False and vice versa. They can’t be both True or both False. This rule of the math logic is one of the oldest and is called “The law of the excluded middle”.

3. Let’s try to construct negation of the several statements.
   a. Number 111111111 is a prime number.
   b. There is nothing on the table.
   c. 0.5 and $\frac{1}{2}$ are not equale.
   d. The area of a rectangle is equal to the product of its length and width.
   e. Sum $18 \cdot 946 + 456$ is divisible by 9.
   f. $45784 > 45784$
   g. $345 < 12345$
   h. All birds can flight.
   i. All marine animals are fish.
   j. Some students like math.
   k. All natural numbers are divisible by 3.
   l. Penguins live on the North Pole.
   m. Polar bears live on the South Pole.

4. Using the law of the excluded middle prove, that the negation of statement was made incorrectly.

	Statement	Negation
1	All cats are gray.	All cats are not gray
2	Some berries are sweet.	Some berries are not sweet.
3	There are 30 days in some months.	There are no 30 days in some months.
4	All birds can fly.	There are no birds that can fly.

Categorical statements are the statements about the relationship between categories or classes of objects. It states whether one category is fully contained with another, is partially contained (there are at least one member of the category) within another, or is completely separate.

“Any (all) natural number is divisible by 3” is a categorical statement and it is a false statement. It is very easy to show: 5 is a natural number and it is not divisible by 3. It means that not any natural number is divisible by 3, some of the them are not disable by 3 and only one such number is enough to prove that the statement is false.

“The sum of any even numbers is an even number” is a categorical statement. The statement is about the category “sum of two arbitrary even numbers”, this category belongs to the set of even numbers. We can ether prove it wrong by showing at least one example of the odd sum of two even number, or prove it true by reasoning. It is not enough to show several examples to prove that this statement is true and of course there are no example to prove it wrong.

Prove. Any even number can be represented as $2k$ (or $2n$) where $k, n \epsilon N$

$$2k + 2n = 2(k + n), \quad k, n \epsilon N$$

So, the sum is divisible by 2, or even number.
How the negation of the categorical statement can be done?
If the statement is about the whole category (all elements of the category) which forms a subset (we can use the set theory formalism here) of another category, the negated statement will show the existence of at least one element of the set, which doesn't belong to the category.

"All birds can fly. " – the statemen is telling us that the whole category (all birds) is belongs to another category, things that can fly. The negation of this statemen should tell us that there is at least one element (one kind of birds) which can't fly. We can formulate it as:
"Some birds can't fly" or as "There are birds that can't fly."

$P =$ "All men are not bald" , $\neg P$ = "Some men are bald" and vice versa:
$P =$ "Some cats are gray", $\neg P$ = "All cats are not gray"