Class work 12. Algebra.

Algebra.

$$\frac{(999^{-1} - 1000^{-1})(999^{-1} + 1000^{-1})}{(1000^{-1} - 999^{-1})^2} =$$

Algebraic identities.

Identities are equalities which are true for any (possible) values of variables.

$(a + b)^2 = (a + b)(a + b) = a^2 + ab + ba + b^2 = a^2 + 2ab + b^2$

$(a + b)(a - b) = a^2 - ab + ba - b^2 = a^2 - b^2$

$(a + b)^3 =$

$(a + b + c)^2 =$

Factorization of a polynomial is a representation of the polynomial as a product of a monomial and polynomial or a product of two or more polynomials.

For example,

$6x^2y - 3y^2x = 3xy(2x - y)$

$2a^4 + 2a^3x^2 + xa + x^3 = 2a^3(a + x^2) + x(a + x^2) = (a + x^2)(2a^3 + x)$

Exercises.

1. Simplify the following expressions (combine like terms):

   $a.$ $7a + (2a + 3b);$ $b.$ $9x + (2y - 5x);$
   $c.$ $(5x + 7a) + 4x;$ $d.$ $(5x - 7a) + 5a;$
   $e.$ $(3x - 6y) - 4y;$ $f.$ $(2a + 5b) - 7b;$
   $g.$ $3m - (5n + 2m);$ $h.$ $6p - (5p - 3a);$

2.
   $a.$ $(x^2 + 4x) + (x^2 - x + 1) - (x^2 - x);$
   $b.$ $(a^5 + 5a^2 + 3a - a) - (a^3 - 3a^2 + a);$
   $c.$ $(x^2 - 3x + 2) - (-2x - 3);$
   $d.$ $(abc + 1) + (-1 - abc);$
   .

3. Factorize the following polynomials:

   $a.$ $x(1 + b) + y(1 + b);$ $f.$ $(a + b)a - b(a + b);$
   $b.$ $m(2k - 3) + 2(2k - 3);$ $g.$ $(x + y)3 - a(x + y);$
   $c.$ $2a(1 - b) - 3(1 - b);$ $h.$ $a(b + 3) - b(3 + b);$
   $d.$ $7x(x - 2y) - 2(2y + x);$ $i.$ $a(a + b) + (a + b);$
   $e.$ $2x(x - 2y) + 3y(x + 2y);$ $j.$ $2x(a - 1) - (a - 1);$

We already know what is GCD and LCM for several natural numbers and we know how to find them.

Exercise:

Find GCD (GCF) and LCM for numbers

a. 222 and 345.
b. $2^2 \cdot 3^3 \cdot 5$ and $2 \cdot 3^2 \cdot 5^2$

Can we apply the same strategy to find CF and CM for algebraic expressions? (In this case the concept of GCD and LCM cannot be applied.) For example, can CF and CM be found for expressions $2x^2y^5$ and $4x^3y^2$? x and y are variables and can't be represented as a product of factors, but they itself are factors, and the expression can be represented as a product:
$2x^2y^5 = 2 \cdot x \cdot x \cdot y \cdot y \cdot y \cdot y \cdot y,$ $\qquad 4x^3y^3 = 2 \cdot 2 \cdot x \cdot x \cdot x \cdot y \cdot y \cdot$
$A = (Factors, 2x^2y^5) = \{2, x, x, y, y, y, y, y\}, B = (Factors, 4x^3y^3) = \{\, 2, 2, x, x, x, y, y\}$

Common devisors are any product of $A \cap B = \{\, 2, x, x, y, y\}$.
What about common multiples? Product of all factors of both numbers (or the product of two numbers) will be the multiple, but minimal common multiple will be the product of the

$$A \cup B = \{2,2, x, x, x, y, y, y, y, y, y\}$$

$$\frac{2x^2y^5}{2 \cdot x^2y^2} = y^3; \qquad \frac{4x^3y^2}{2 \cdot x^2y^2} = 2x;$$

$$\frac{4x^3y^5}{2 \cdot x^2y^5} = 2x; \qquad \frac{4x^3y^5}{4 \cdot x^3y^3} = y^2;$$

Algebraic fraction are expression are a fraction $\frac{A}{B}$ $(B \neq 0)$ whose numerator and denominator are algebraic expressions (not necessarily polynomials). For example:

$$\frac{3x^2 + y}{y^2 - 5x + 2}; \qquad \frac{\frac{1}{x} - 3}{y + \frac{1}{y}}$$

$$\frac{A}{1} = A; \qquad \frac{A}{B} = \frac{A \cdot C}{B \cdot C} \quad (C \neq 0); \qquad -\frac{A}{B} = \frac{-A}{B} = \frac{B}{-A}$$

4. Add fractions:
   Example:
   $$\frac{2}{x^2a} + \frac{3}{a^2x} = \frac{2a}{a^2x^2} + \frac{3x}{a^2x^2} = \frac{2a + 3x}{a^2x^2}$$

$a.\quad \frac{1}{a}+\frac{1}{b};$ $\qquad b.\quad \frac{2}{x}-\frac{3}{y};$ $\qquad c.\frac{x}{a}+\frac{y}{b};$

$d.\quad \frac{5a}{7}-\frac{b}{x};$ $\qquad b.\quad \frac{1}{2a}-\frac{1}{3};$ $\qquad c.\frac{1}{a}-\frac{1}{bc};$

5. Euler formula for prime numbers:
   $n^2 - n + 41$ is a prime number for any $n \in N$. Prove or disapprove it.

6. Compute:

$$\frac{10^2 + 11^2 + 12^2 + 13^2 + 14^2}{365}$$