Class work 11. Algebra.

Algebra.

Monomials.

Monomial is a product of variables in nonnegative integer power and a number, which is called a coefficient. For example: $xy^3 6,\ 56,\ 3c^5\ d^{10}, 2x3y5$.
Two monomials are equal if their difference is only on the order of factors.
Monomial is equal to 0 if one of the factors is 0.
Usually, monomials are written in the following form: first goes a coefficient (only one number), then the variable with the highest power and so on…Example above $6y^3x, 56, 3d^{10}c^5, 30xy.$
Degree of a monomial is the sum of all exponents of variables. The degree of $6y^3x$ is 4 $(1 + 3 = 4)$.
Several monomials can be added together and/or multiply.

$$5x^2m^3 \cdot 7m^2y^3 = 35x^2m^5y^3$$

Polynomials.

We can add together a few monomials and get a polynomial:

$$A = 6y^3x + 56 + 3d^{10}c^5 + 30xy$$

The degree of a polynomial is the highest of the degrees of its monomials (individual terms) with non-zero coefficients. The term order has been used as a synonym of degree but, nowadays, may refer to several other concepts. For example, the polynomial $7x^2y^3 + 4x - 9$ which can also be expressed as $7x^2y^3 + 4x^1y^0 - 9x^0y^0$ has three terms. The first term has a degree of 5 (the sum of the powers 2 and 3), the second term has a degree of 1, and the last term has a degree of 0. Therefore, the polynomial has a degree of 5, which is the highest degree of any term.

To determine the degree of a polynomial that is not in standard form (for example: $(x + 1)^2 - (x - 1)^2$, one has to put it first in standard form by expanding the products (by distributivity) and combining the like terms; $(x + 1)^2 - (x - 1)^2 = 4x$ is of degree 1, even though each summand has degree 2.
Polynomials can be added together

$$(7x^2y^3 + 4xy^2 - 6) + (3x^2y^3 - 2x^5y^2) = 7x^2y^3 + 4xy^2 - 6 + 3x^2y^3 - 2x^5y^2$$
$$= 10x^2y^3 + 4xy^2 - 2x^5y^2 = -2x^5y^2 + 10x^2y^3 + 4xy^2$$

Multiplication of polynomials.

How to multiply polynomials?

$$(a + b) \cdot (c + d) = ?$$

We know how to multiply an expression by a number using the distributive property: $a \cdot (b + c) = ab + ac$. What should we do to multiply one expression by another? To simplify the problem let's do the substitution, $a + b = u$ and use the distributive property:

$$(a + b) \cdot (c + d) = u(c + d) = uc + ud$$

This new expression is not exactly the result what we are looking for; so, we need to put back $(a + b)$ instead of u:

$$uc + ud = (a + b)c + (a + b)d$$

To get the final result let's use the distributive property again:

$$(a + b)c + (a + b)d = ac + bc + ad + bd$$

More polynomial multiplications:

$$(2x^2 + 3y^3) \cdot (3x^3 + y^5) = 2x^2 \cdot 3x^3 + 2x^2 \cdot y^5 + 3y^3 \cdot 3x^3 + 3x^3 \cdot y^5$$
$$= 6x^5 + 2x^2y^5 + 9x^3y^3 + 3x^3y^5$$

Algebraic identities is an expression which is true for any values of variables.

A few similar identities are very useful:

1. $(a + b)^2 = a^2 + 2ab + b^2 = (-a - b)^2$
2. $(a - b)^2 = a^2 - 2ab + b^2$
3. $(a - b)(a + b) = a^2 - b^2$
4. $(a + b + c)^2 = a^2 + b^2 + c^2 + 2ab + 2bc + 2ca$
5. $(a + b - c)^2 = a^2 + b^2 + c^2 + 2ab - 2bc - 2ca$
6. $(a - b + c)^2 = a^2 + b^2 + c^2 - 2ab - 2bc + 2ca$
7. $(-a + b + c)^2 = a^2 + b^2 + c^2 - 2ab + 2bc - 2ca$
8. $(a - b - c)^2 = a^2 + b^2 + c^2 - 2ab + 2bc - 2ca$
9. $(a + b)^3 = a^3 + b^3 + 3ab(a + b)$
10. $(a - b)^3 = a^3 - b^3 - 3ab(a - b)$
11. $a^3 + b^3 = (a + b)^3 - 3ab(a + b)$
    $= (a + b)(a^2 - ab + b^2)$
12. $a^3 - b^3 = (a - b)^3 + 3ab(a - b)$
    $= (a - b)(a^2 + ab + b^2)$
13. $a^3 + b^3 + c^3 - 3abc = (a + b + c)(a^2 + b^2 + c^2 - ab - bc - ca)$
    if $a + b + c = 0$ then $a^3 + b^3 + c^3 = 3abc$

Exercises.

1. Which of the following are monomials?

a) a;	d) $a + b$;	g) ba;	j) $b2c$;
b) $\frac{ab}{a+b}$;	e) $\frac{ax}{b}$;	h) $\frac{3}{4}xy$;	k) $7a - 3$;
c) $-1,(26)$;	f) $(a - b) \cdot 3$;	i) $\frac{p}{b}axy$;	l) 0?

2. Simplify the following expression (combine like terms, think about which terms you can add together and which you can't):

$$\left(\frac{1}{7}klm^2 - \frac{4}{3}kl^2m + 7klm\right) + \left(-\frac{3}{21}klm^2 + \frac{4}{9}kl^2m - 5klm\right);$$

3. Simplify the following expressions (rewrite the expressions without parenthesis, combine like terms);

Example:

$$(2x + 3) \cdot (x + 7) = 2xx + 2x \cdot 7 + 3x + 3 \cdot 7 = 2x^2 + 10x + 21$$

$(x+5)(x+y+3);$

$(k-1+d)(k-d);$

$\frac{2}{3}+2x\left(\frac{1}{2}-\frac{1}{3}y\right)-x-\frac{1}{3}(2-2xy);$

$2x^2(x+y)-3x^2(x-y);$

4. Factor out the common factor;

а) a^2+ab; **б)** x^2-x; **в)** $a+a^2$;
г) $2xy-x^3$; **д)** b^3-b^2; **е)** a^4+a^3b;
ж) $x^2y^2+y^4$; **з)** $4a^6-2a^3b$; **и)** $9x^4-12x^2y^4$.