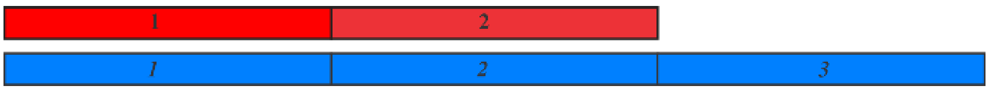
Class work 10. Algebra.

Algebra.

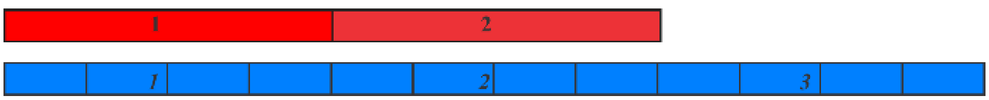
Irene has a total of 1686 red, blue and green balloons for sale. The ratio of the number of red balloons to the number of blue balloons was 2:3. After Irene sold 3/4 of the blue balloons, 1/2 of the green balloons and none of the red balloons, she has 922 balloons left. How many blue balloons did Irene have at first?

Step 1. For each 2 red balloons there are three blue balloons, so we can show all red and blue balloons as:



We took as "unit" a half of the red balloons. The number of blue balloons is $\frac{3}{2}$ times more than number of red balloons (or three times as much as a half of the red ones)

Step 2. $\frac{3}{4}$ of the blue balloons were sold. We can't divide 3 "units" into 4 parts, without getting fractions. So, let's find LCM of 3 and 4 and divide the number of blue balloons into 12 parts.



Step 3. Let's compare the number of sold and leftover balloons.

922
$1686 - 922 = 764$

Number of sold and unsold green balloons are the same, red balloons are all left, as well as $\frac{1}{4}$ of blue balloons. As we can see 2 small "units" of blue balloons are $922 - 764 = 158$, or one such "unit" is 79. Total amount of blue balloons is $158 \cdot 6 = 948$. The number of red balloons is

$$\frac{2}{3} \cdot 948 = 632.$$

Number of dreen ones is $1686 - (632 + 948) = 106$

Can we solve the problem by writing equations?

Let's try.

$G + B + R = 1686$

$3R = 2B$

$\frac{1}{2}G + R + \frac{1}{4}B = 922$

$\frac{1}{2}G + R + \frac{1}{4}B - \left(\frac{1}{2}G + \frac{3}{4}B\right) = 922 - 764$

$R - \frac{1}{2}B = 158$

$\frac{2}{3}B - \frac{1}{2}B = 158 \quad \Rightarrow \quad \left(\frac{4}{6} - \frac{3}{6}\right)B = 79 \quad \Rightarrow \quad B = 6 \cdot 158$

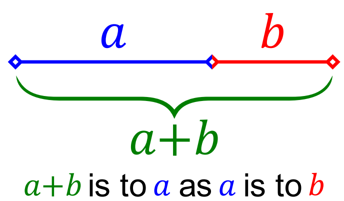
Famous ratios.

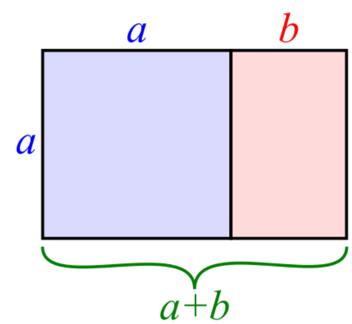
i. Let's measure the circumference and the diameter of a circle.

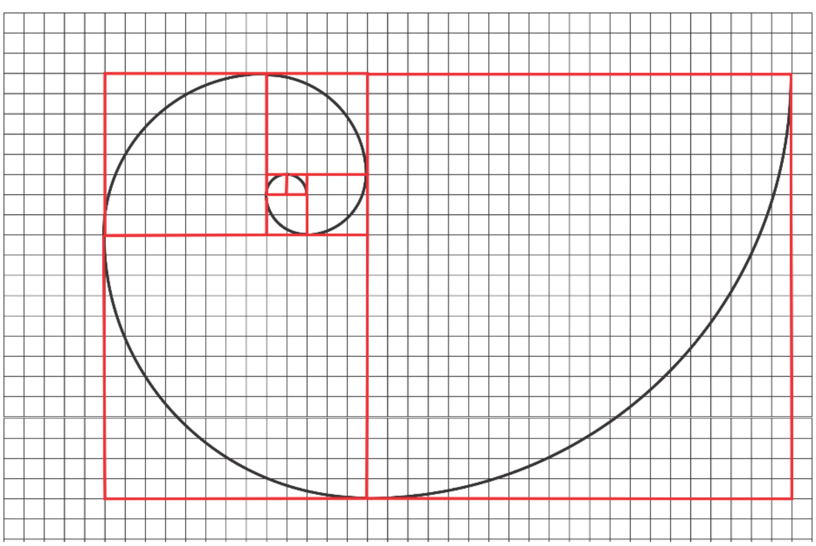
$\frac{l}{d} = \pi$

ii. Golden ratio:

$\frac{a+b}{a} = \frac{a}{b} \cong 1.618$



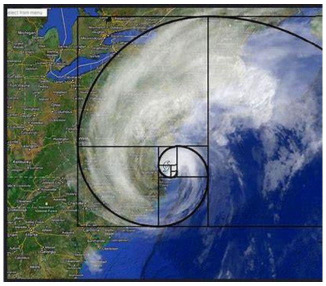




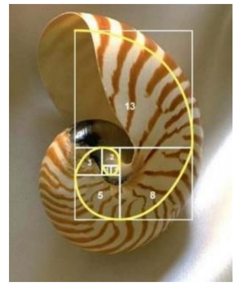
Fibonacci sequence:

$1, 1, 2, 3, 5, 8 \ldots ..$

$F_n = F_{n-1} + F_{n-2}$



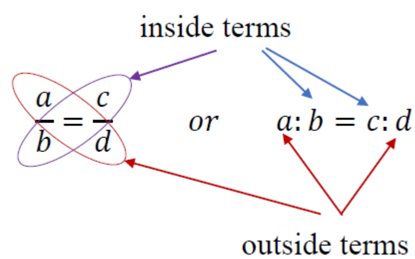
To cook a raspberry jam according to recipe I need to combine three cups of berries and 2 cups of sugar, or for each 3 cups of raspberries go 2 cups of sugar; ratio of raspberries and sugar (in volume) is $3:2$. If I bought 27 cups of raspberries, how many cups of sugar do I need to put to my jam?



$$\frac{3}{2} = \frac{27}{x}$$

Two ratios which are equal form a proportion.

Proportions have several interesting features.



1. The product of inside and outside terms are equal.

$$\frac{a}{b} = \frac{c}{d} \quad \Leftrightarrow \quad a \cdot d = b \cdot c$$

It can be easily shown:

$$\frac{a}{b} = \frac{c}{d} \Leftrightarrow \frac{adb}{b} = \frac{cdb}{d} \Leftrightarrow ad = cb$$

2. Also, two inverse ratios are equal:

$$\frac{a}{b} = \frac{c}{d} \quad \Leftrightarrow \quad \frac{b}{a} = \frac{d}{c}$$

Indeed:

$$\frac{a}{b} = \frac{c}{d} \quad \Leftrightarrow \quad a \cdot d = b \cdot c \quad \Leftrightarrow \quad \frac{ad}{ac} = \frac{bc}{ac} \quad \Leftrightarrow \quad \frac{d}{c} = \frac{b}{a}$$

3. Two outside terms can be switched:

$$\frac{a}{b} = \frac{c}{d} \quad \Leftrightarrow \quad \frac{d}{b} = \frac{c}{a}$$

$$\frac{a}{b} = \frac{c}{d} \quad \Leftrightarrow \quad a \cdot d = b \cdot c \quad \Leftrightarrow \quad \frac{ad}{ab} = \frac{bc}{ab} \quad \Leftrightarrow \quad \frac{d}{c} = \frac{b}{a}$$

4. Two inside terms can be switched as well.

$$\frac{a}{b} = \frac{c}{d} \quad \Leftrightarrow \quad \frac{a}{c} = \frac{b}{d}$$

5. Also, several other new proportion can be created.

$$\frac{a}{b} = \frac{c}{d} \quad \Leftrightarrow \quad \frac{a \pm b}{b} = \frac{c \pm d}{d}$$

(the sign $\pm$ is used to show that both, addition and subtraction, can be used)

Let's prove one of the statements:

$$\frac{a}{b}=\frac{c}{d} \quad \Leftrightarrow \quad \frac{a}{b}+1=\frac{c}{d}+1 \;\Leftrightarrow\; \frac{a}{b}+\frac{b}{b}=\frac{c}{d}+\frac{d}{d} \quad \Leftrightarrow \quad \frac{a+b}{b}=\frac{c+d}{d}$$

6. Another proportion:

$$\frac{a}{b}=\frac{c}{d} \quad \Leftrightarrow \quad \frac{a+c}{b+d}=\frac{c}{d}=\frac{a}{b}$$

It can be proved as follow:

$$\frac{a+c}{b+d}=\frac{a\left(1+\frac{c}{a}\right)}{b\left(1+\frac{d}{b}\right)}$$

We know form (4) that

$$\frac{c}{a}=\frac{d}{b}$$

$$\frac{a+c}{b+d}=\frac{a\left(1+\frac{c}{a}\right)}{b\left(1+\frac{d}{b}\right)}=\frac{d}{b}$$

Going back to the jam problem above. We got the simple equation

$$\frac{3}{2}=\frac{27}{x}$$

It can be solved easily using the property of proportion

$$3x=27\cdot 2$$

$$x=\frac{27\cdot 2}{3}=\frac{3\cdot 9\cdot 2}{3}=18$$

1. Three solutions of salt with concentration 10%, 15%, and 30% (it means that in the solution there are 10% (or 15%, or 30%) of the total mass is NaCl and 90% (or 85%, or 70%) is water) are mixed together. The mass of the first solution is 180g, mass of the second solution is twice as the mass of the first solution, and the mass of the third solution is 100 g. greater than the mass of the second solution. What is the concentration of the mixture?
2. Solve the following equations (hint: use the property of proportions):

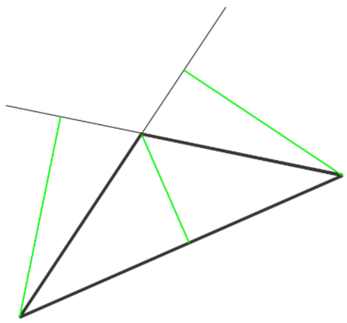
$$a.\ \frac{x}{7.2}=\frac{1\frac{1}{9}}{0.25}; \qquad b.\ \frac{2\frac{1}{3}}{0.6x}=\frac{2.5}{1\frac{2}{7}}; \qquad c.\ \frac{\frac{7}{12}}{0.14}=\frac{50x}{4.8}; \qquad d.\ \frac{1\frac{3}{17}}{13.75}=\frac{2\frac{2}{11}}{3x}$$

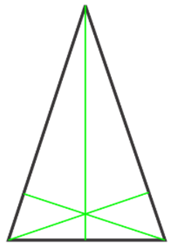
3. Dry cranberries contain 25% of water. How much water should be evaporated from 5 kg of fresh cranberries to get dry cranberries, if fresh cranberries contain 85% of water?

Geometry.

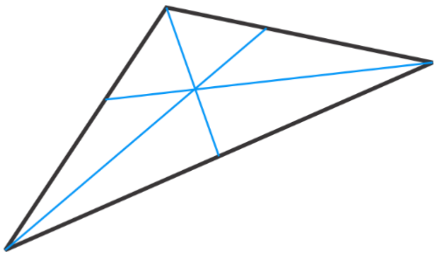
Special segments of a triangle.

From each vertices of a tringle to the opposite side 3 special segment can be constructed.





An **altitude** of a triangle is a straight line through a vertex and perpendicular to (i.e. forming a right angle with) the opposite side. This opposite side is called the *base* of the altitude, and the point where the altitude intersects the base (or its extension) is called the *foot* of the altitude.



An **angle bisector** of a triangle is a straight line through a vertex which cuts the corresponding angle in half.

A **median** of a triangle is a straight line through a vertex and the midpoint of the opposite side, and divides the triangle into two equal areas.