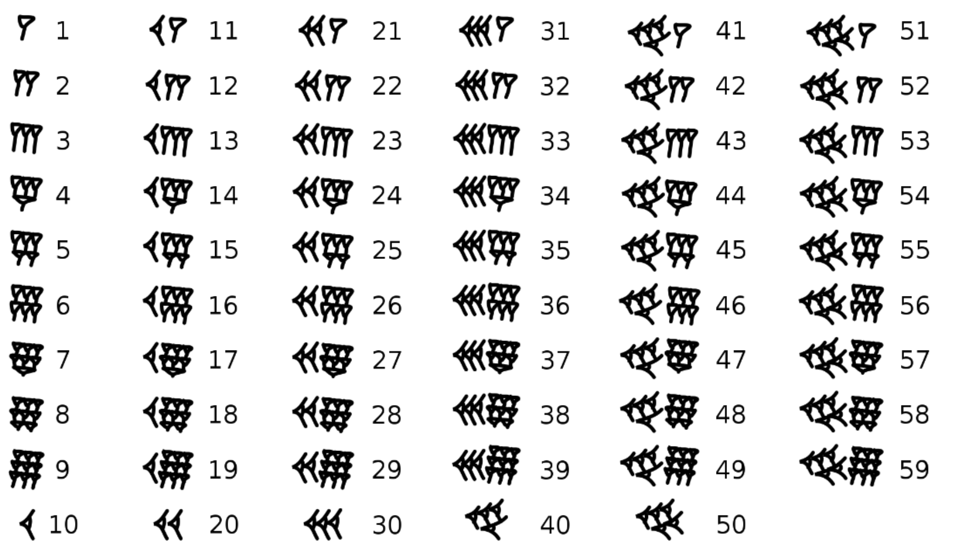
Class work 7. Algebra.

Algebra.

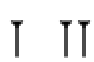
Numeral systems.

Over the long centuries of human history, m*Type equation here.*any different numeral systems have appeared in different cultures. The oldest systems weren’t a place-valued system, but sometimes use the position of the “digit” to show units. For example, the Babylonian system (form about 2000 BC) used only two symbols to write any number between 1 and 60:

 to count units and  to count tens.



By Josell7 - File:Babylonian_numerals.jpg, CC BY-SA 4.0, https://commons.wikimedia.org/w/index.php?curid=9862983

Number 62 was shown as  which means one time 60 and 2. The use of this sexagesimal (60-based) system is still visible, we have 60 minutes in one hour, 60 seconds in a minute, 360° for the full turn around.

Another very well known numeral system is roman, it was used for thousands of years and in some cases is still used now as well. It's a “decimal”, 10 based system, but the symbols (letters) are used in an unusual way.

Symbol	I	V	X	L	C	D	M
Value	1	5	10	50	100	500	1000

For example, 4 is one less than 5, so 4 can be written as IV. Same principle of subtractive notation is used for 9 -> IX, 40 and 90 -> XL and XC, 400 and 900 -> CD and CM

1	2	3	4	5	6	7	8	9	10
I	II	III	IV	V	VI	VII	VIII	IX	X

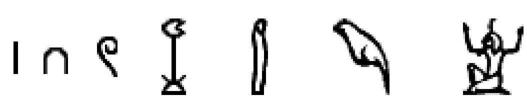
Some other examples:

- 29 = XX + IX = **XXXIX**.
- 347 = CCC + XL + VII = **CCCXLVI**.
- 789 = DCC + LXXX + IX = **DCCLXXXIX**.
- 2,421 = MM + CD + XX + I = **MMCDXXI**.

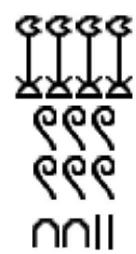
Any missing place (represented by a zero in the place-value equivalent) is omitted, as in Latin (and English) speech:

- 160 = C + LX = **CLX**
- 207 = CC + VII = **CCVII**
- 1,009 = M + IX = **MIX**
- 1,066 = M + LX + VI = **MLXVI**

Egyptian numeral system was decimal (10-based), similar to the one we use now, but also wasn't placed-valued and didn't use the position of the symbol in any way.

Value	1	10	100	1,000	10,000	100,000	1 million, or many
Hieroglyph (image)							

To write a number, you need to draw the symbol of units, tens, and so on as many times as there are units, tens, and so on. For example, the number 4622 should be written as (order in which all the symbols are written is not important).:



The most familiar numeral system for us is our decimal place-valued system, were the position of the digit defines its value. In the extended form we can right any number as a sum of 10^n multiplied by a numbers.

$$2345 = 2000 + 300 + 40 + 5 = 1000 \cdot 2 + 100 \cdot 3 + 10 \cdot 4 + 5$$
$$= 10^3 \cdot 2 + 10^2 \cdot 3 + 10^1 \cdot 4 + 10^0 \cdot 5$$

This system can be extended to fractional parts of the number:

$$2345.23 = 10^3 \cdot 2 + 10^2 \cdot 3 + 10^1 \cdot 4 + 10^0 \cdot 5 + \frac{1}{10} \cdot 2 + \frac{1}{100} =$$
$$= 10^3 \cdot 2 + 10^2 \cdot 3 + 10^1 \cdot 4 + 10^0 \cdot 5 + 10^{-1} \cdot 2 + 10^{-2} \cdot 3$$

Can non-decimal place-value system be created? Foe example with base 5?

Let see, how we can create this kind of system (we use our normal digits).

Num_{10}	1	2	3	4	5	6	7	8	9	10
Num_5	1	2	3	4	10	11	12	13	14	20

11	12	13	14	15	16	17	18	19	20
21	22	23	24	30	31	32	33	34	40

21	22	23	24	25	26	27	28	29	30
41	42	43	44	100	101	102	103	104	110

We only have 5 digits (0, 1, 2, 3, 4), and 4 first "natural" numbers in such system will be represented just with digits. Number 5 then should be shown as a 2-digit number, with fist digit 1 (place – value equal to 5^1) and 0 of "units". Any number is now written if the form

$$5^3 \cdot m + 5^2 \cdot n + 5^1 \cdot k + 5^0 \cdot p$$

$$33 = 25 + 5 + 3 = 5^2 \cdot 1 + 5^1 \cdot 1 + 3 \to 113$$

$$195 = 125 + 2 \cdot 25 + 20 = 5^3 + 5^2 \cdot 2 + 5^1 \cdot 4 + 0 \to 1240$$

And vice versa, if we need transform the number form 5-base to decimal system:

$$2312 \to 5^3 \cdot 2 + 5^2 \cdot 3 + 5^1 \cdot 1 + 5^0 \cdot 2 = 250 + 75 + 5 + 2 = 332$$

Let's try to introduce a new digit $\vdash$ for 10 and then create an 11 based system.

$$11^2 = 121, \qquad 11^3 = 1331$$

$$890 = 847 + 33 + 10 = 121 \cdot 7 + 11 \cdot 3 + 10 = 11^2 \cdot 7 + 11^1 \cdot 3 + 11^0 \cdot 10 \to 73\vdash$$

$$4\vdash 2 \to 11^2 \cdot 4 + 11^1 \cdot 10 + 11^0 \cdot 2 = 484 + 110 + 2 = 596$$

There is another very important place-value system: binary system, base 2 system where only two digits exist; 0, and 1.

Num$_{10}$	1	2	3	4	5	6	7	8	9	10
Num2	1	10	11	100	101	110	111	1000	1001	1010

In this system each number is represented as

$2^3 \cdot (0,1) + 2^2 \cdot (0,1) + 2^1 \cdot (0,1) + 2^0 \cdot (0,1)$

$11 = 8 + 2 + 1 = 2^3 \cdot 1 + 2^2 \cdot 0 + 2^1 \cdot 1 + 2^0 \cdot 1 \rightarrow 1011$

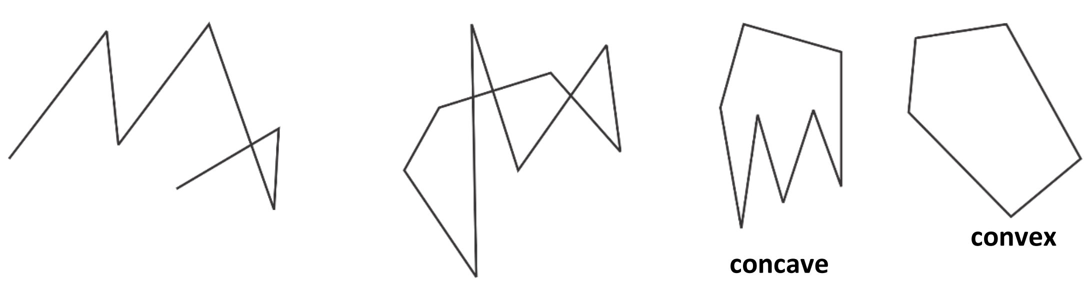
$$75 = 64 + 8 + 2 + 1 = 2^6 + 2^3 + 2 + 1$$
$$= 2^6 \cdot 1 + 2^5 \cdot 0 + 2^4 \cdot 0 + 2^3 \cdot 1 + 2^2 \cdot 0 + 2^1 \cdot 1 + 2^0 \cdot 1 \rightarrow 1001011$$

1. How to arrange 127 dollar bills in seven wallets so that any amount from 1 to 127 dollars could be issued without opening the wallets?
2. Robert thought of a number not less than 1 and not more than 1000. Julia is allowed to ask only such questions to which Robert can answer “yes” or “no” (Robert always tells the truth). Can Julia determine the hidden number in 10 questions?
3. There is a bag of sugar, a scale and a weight of 1 g. Is it possible to measure 1 kg of sugar in 10 weights?

Polygons.

Draw a chain of segments, so that the last point of one segment is a first point of the next, and two consecutive points don’t lie on the same line.

Draw such chain so that the last point of the last segment is the first point of the first one. We got a closed broken line. Is this a sufficient condition to get a polygon?



In geometry, a **polygon** is a plane figure that is bounded by a finite chain of straight line segments closing in a loop to form a closed chain. These segments are called its *edges* or *sides*, and the points where two edges meet are the polygon's *vertices* (singular: vertex) or *corners*. The interior of the polygon is sometimes called its *body*. An ***n*-gon** is a polygon with *n* sides; for example, a triangle is a 3-gon.

- *What is the difference between convex and concave polygons?*

  The simplest polygon is a triangle.

  Draw a triangle. Measure its angles. Add them. How much did you get?

1. Draw the isosceles
   a. right triangle
   b. acute triangle
   c. obtuse triangle

2. Draw a triangle with sides 3 cm, 5 cm and the angle between them 50°.

3. Draw a triangle with angles 30° and 50° and the side between them 7 cm. Do we need another information to construct a triangle?

4. What is a circle? Draw a cercle (use compass)

Circle is the set of all points in a plane that are at a given distance from a given point, the center.