Class work 4. Algebra.

Algebra.

Prime factorization or integer **factorization** of a number is the determination of the set of **prime** numbers which multiply together to give the original integer. It is also known as **prime** decomposition.

168	*2*		*180*	*2*
84	*2*		*90*	*2*
42	*2*		*45*	*3*
21	*3*		*15*	*3*
7	*7*		*5*	*5*
1			*1*	

Prime factorization process:

Prime factors of 168 are $2, 2, 2, 3, 7$ and prime factors of 180 are $2, 2, 3, 3, 5,$

$$2 \times 2 \times 2 \times 3 \times 7 = 168; \quad 2 \times 2 \times 3 \times 3 \times 5 = 180$$

Eratosthenes proposed a simple algorithm for finding prime numbers. This algorithm is known in

mathematics as the Sieve of Eratosthenes.

In mathematics, the sieve of Eratosthenes, one of a number of prime number sieves, is a simple, ancient algorithm for finding all prime numbers up to any given limit. It does so by iteratively marking as composite, *i.e.*, not prime, the multiples of each prime, starting with the multiples of 2.

~~1~~	2	3	~~4~~	5	~~6~~	7	~~8~~	~~9~~	~~10~~
11	~~12~~	13	~~14~~	~~15~~	~~16~~	17	~~18~~	19	~~20~~
~~21~~	~~22~~	23	~~24~~	~~25~~	~~26~~	~~27~~	~~28~~	29	~~30~~
31	~~32~~	~~33~~	~~34~~	~~35~~	~~36~~	37	~~38~~	~~39~~	~~40~~
41	~~42~~	43	~~44~~	~~45~~	~~46~~	47	~~48~~	~~49~~	~~50~~
~~51~~	~~52~~	53	~~54~~	~~55~~	~~56~~	~~57~~	~~58~~	59	~~60~~
61	~~62~~	~~63~~	~~64~~	~~65~~	~~66~~	67	~~68~~	~~69~~	~~70~~
71	~~72~~	73	~~74~~	~~75~~	~~76~~	~~77~~	~~78~~	79	~~80~~
~~81~~	~~82~~	83	~~84~~	~~85~~	~~86~~	~~87~~	~~88~~	89	~~90~~
~~91~~	~~92~~	~~93~~	~~94~~	~~95~~	~~96~~	97	~~98~~	~~99~~	~~100~~

fundamental theorem of arithmetic:

Any natural number greater than 1 either is a prime number or can be represented as a product of prime numbers and such representation is unique.

For example:

$$1200 = 5 \cdot 2 \cdot 5 \cdot 2 \cdot 3 \cdot 2 \cdot 2$$

The theorem says two things, for this example: first, that 1200 can be represented as a product of primes, and second, that no matter how this is done, there will always be exactly four 2s, one 3, two 5s, and no other primes in the product. This theorem is a main reason why 1 is not considered a prim number.

LCM and GCF (GCD).

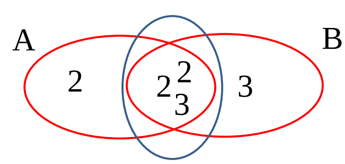
Each natural number is a prime number or can be represented as product of a unique set of prime numbers (see above, fundamental theorem of arithmetic). How we can find all divisors of a number? If the number is prime, there is no other divisors, but itself. If the number is not prime – each prime factor is a divisor, as well as product of all possible combinations (subsets of the set of prime factors). For example, number 24 has a prime representation $24 = 2 \cdot 2 \cdot 2 \cdot 3$, therefore 4, 8, 6, and 12, as well as 24, will be also the divisors of 24. Any two (or more) natural numbers can have common divisors (such that both numbers can be divided by evenly), or, in case that there are no such common divisors, they are called mutually prime. For example, 9 and 20:

$$9 = 3 \cdot 3, \qquad 20 = 2 \cdot 2 \cdot 5$$

These two numbers are not prime, but because they don't have common divisors, they are mutually prime.
How to find common factor? Take a look at the prime factorization of numbers 24 and 36:

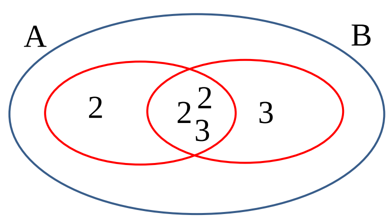
$$24 = 2 \cdot 2 \cdot 2 \cdot 3$$
$$36 = 2 \cdot 2 \cdot 3 \cdot 3$$

Both numbers have common factors, so they both can be divided two times by 2 and by 3, and by the product of any combinations of these three numbers. The greatest divisor (greatest common factor will be the product of all common factors. This can be also be represented as the Venn diagrams of the sets of prime factors of numbers 24, 36, and the intersection of these two sets. Set $\text{A} = (P, 24), \quad \text{B} = (P, 36)$



Multiple of a number a is any number, which is divisible by a. $(M = na,\ n \in N)$. For two numbers, a and b common multiple is a number which is divisible by both, a and b. One of the common multiples is the product of a and b, but it's not necessarily the smallest one.

$$24 = 2 \cdot 2 \cdot 2 \cdot 3$$
$$36 = \quad 2 \cdot 2 \cdot 3 \cdot 3$$



The product of the common prime factors together with the remaining prime factors from both numbers will be divisible by both numbers, and will be the smallest multiple. In terms of set theory, this will be the product of all elements of the unity of sets A an B.

Exercises:

1. $a + 1$ is divisible by 3. Prove that $4 + 7a$ is divisible by 3 as well.

2. $2 + a$ and $35 - b$ are both divisible by 11. Prove that $a + b$ is divisible by 11 as well.

3. Even or odd number will be the sum
   $1 + 2 + 3 + \ldots + 10$
   $1 + 2 + 3 + \ldots + 100$
   $1 + 2 + 3 + \ldots + 100$

4. Can you say which of the following statements are true and which are false?
   a. If the natural number is divisible by 3 and 5, it's divisible by 15
      (if $a \vdots 3$ and $a \vdots 5 \Rightarrow a \vdots 15$)
   b. If the natural number is divisible by 15, it's divisible by 3 and 5.
      (if $a \vdots 15 \Rightarrow$ $a \vdots 3$ and $a \vdots 5$)
   c. If natural number b is even, then $3b$ is divisible by 6. (if $b \vdots 2$, then $3b \vdots 6$)

5. Can the expression below be a true statement, if letters are replaced with numbers from 1 to 9 (different letters correspond to different numbers).

$$f \cdot l \cdot y = i \cdot n \cdot s \cdot e \cdot c \cdot t$$

6. Dunno boasted ability to multiply in the mind. To test it, Doono suggested writing some number, multiplying its digits, and saying the result. "2178," Dunno immediately blurted out, only having had time to write down the number. "It cannot be," - replied, thinking, Doono. How did he detect the error without knowing the source number?

7. Two buses leave from the same bus station following two different routes. For the first one it takes 48 minutes to complete the roundtrip route. For the second one it takes 1 hour and 12 minutes to complete the round-trip route. How much time will it take for the buses to meet at the bus station for the first time after the have departed for their routes at the same time?

8. A florist has 36 roses, 90 lilies, and 60 daisies. What is largest amount of bouquets he can create from these flowers evenly dividing each kind of flowers between them?

9. Find the GCF for the following prime factorized numbers:

a. $a = 2 \cdot 3 \cdot 5$, $b = 2 \cdot 3 \cdot 11$

b. $a = 3 \cdot 3 \cdot 7 \cdot 7$, $b = 2 \cdot 2 \cdot 2 \cdot 5$

c. $a = 2 \cdot 2 \cdot 3 \cdot 3 \cdot 3 \cdot 5 \cdot 7$, $b = 2 \cdot 3 \cdot 3 \cdot 5 \cdot 5 \cdot 11$

10. Is the number a is divisible by number b?

a. $a = 2 \cdot 2 \cdot 2 \cdot 5 \cdot 11$, $b = 2 \cdot 2 \cdot 11$

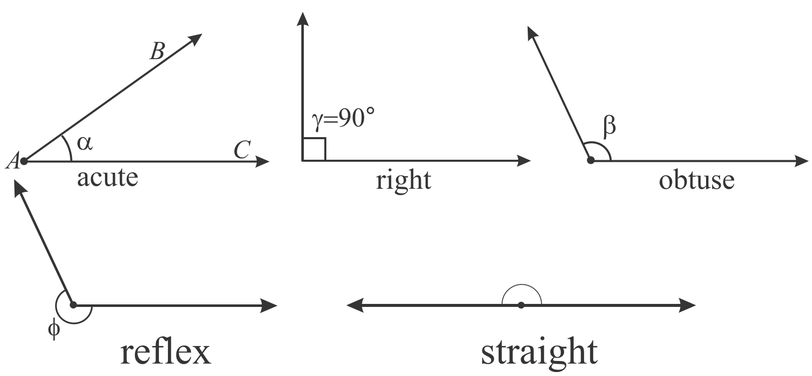
b. $a = 3 \cdot 3 \cdot 5 \cdot 13$, $b = 2 \cdot 13$

c. $a = 2 \cdot 3 \cdot 5 \cdot 5 \cdot 17$, $b = 2 \cdot 3 \cdot 3 \cdot 5$

Geometry.

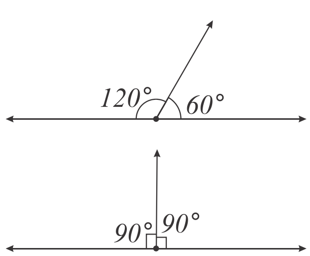
An angle is the figure formed by two **rays**, called the sides of the angle, sharing a common endpoint, called the **vertex** of the angle.

Angles notations are usually three capital letters with vertex letter in the middle or small Greek letter: ∠ABC, α. Measure of the angle is the amount of rotation required to move one side of the angle onto the other. As the angle increases, the name changes:

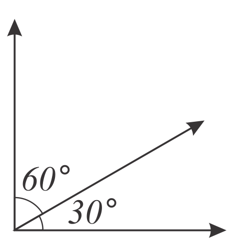


Strait angle is formed by two rays on the same strait line. Straight angle has a measure of 180°.

Two angles are called adjacent if they have common vertex and a common side. If two adjacent angles combined form straight angle they are called supplementary; if they form right angle than they are called complementary.



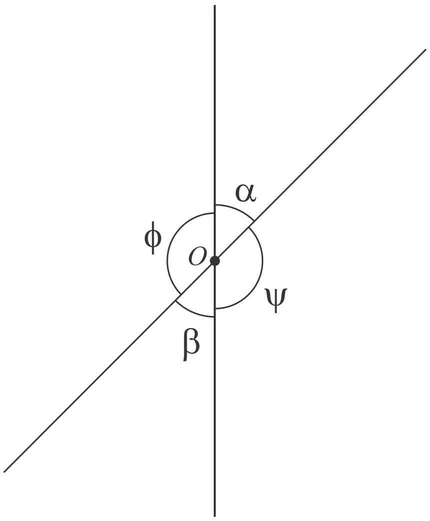
supplementary



complementary

An angle which is supplementary to itself we call right angle. Lines which intersect with the right angle we call perpendicular to each other.

When two straight lines intersect at a point, four angles are formed. A pair of angles opposite each other formed by two intersecting straight lines that form an "X"-like shape, are called vertical angles, or opposite angles, or vertically opposite angles.



α and β and ϕ and ψ are 2 pairs of vertical angles.

Vertical angles theorem:

Vertical angles are equal.

In mathematics, a **theorem** is a statement that has been proven on the basis of previously established statements. According to a historical legend, when Thales visited Egypt, he observed that whenever the Egyptians drew two intersecting lines, they would measure the vertical angles to make sure that they were equal. Thales concluded that one could prove that vertical angles are always equal and there is no need to measure them every time.

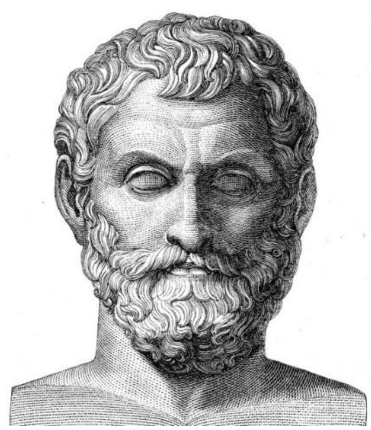
Proof:

$\angle\phi + \angle\alpha = 180°$ because they are supplementary by construction.

$\angle\phi + \angle\beta = 180°$ because they are supplementary also by construction.

$\Rightarrow \angle\alpha = \angle\beta$, therefore we proved that if 2 angles are vertical angles then they are equal. Can we tell that invers is also the truth? Can we tell that if 2 angles are equal than they are vertical angels?

(**Thales of Miletus** 624-546 BC was a Greek philosopher and mathematician from Miletus. Thales attempted to explain natural phenomena without reference to mythology. Thales used geometry to calculate the heights of pyramids and the distance of ships from the shore. He is the first known individual to use deductive reasoning applied to geometry, he also has been credited with the discovery of five theorems. He is the first known individual to whom a mathematical discovery has been attributed (Thales theorem).



Exercises.

11. Draw 3 different angles, measure them with a protractor.
12. Draw angles with the measures $72°,\ 155°, 90°$. Use ruler and protractor.
13. 4 angles are formed at the intersection of 2 lines. One of them is 30°. What is the measure of 3 others?
14. Draw 2 angles in such way that they intersect
    a. by a point
    b. by a segment
    c. by a ray
    d. don’t intersect at all.