Class work 2. Algebra.

a.
```
   SEND
+  MORE
  MONEY
```

b.
```
    EAT
+  THAT
  APPLE
```

c.
```
  CIRCLE
  CIRCLE
+ CIRCLE
  SPHERE
```

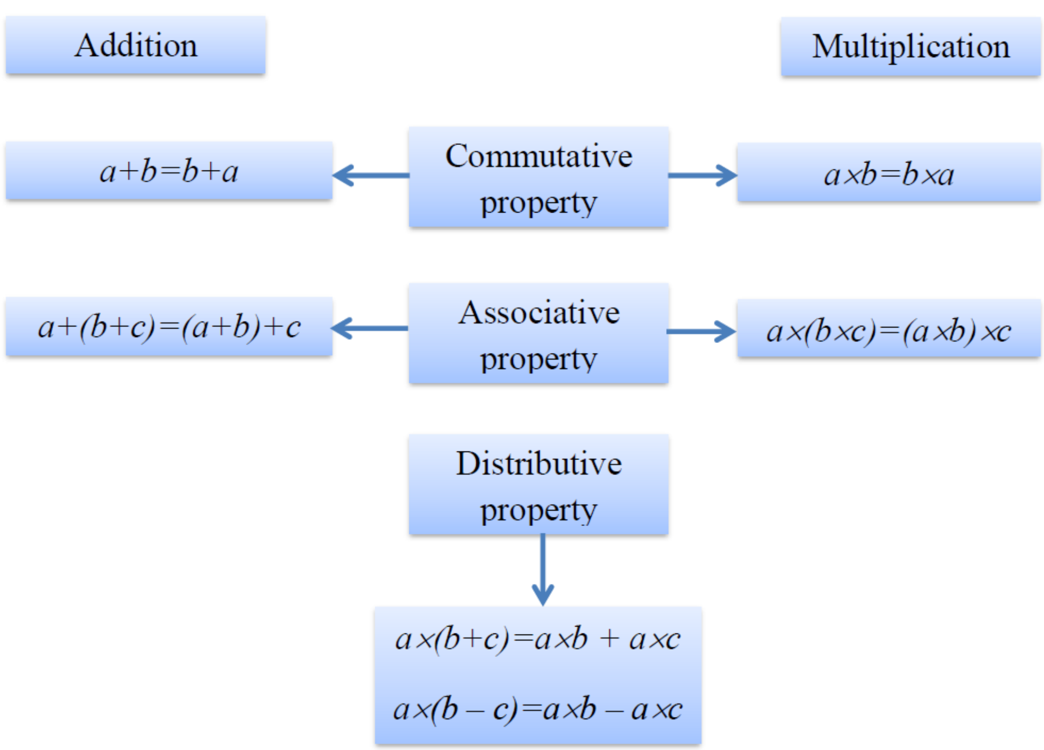
d.
```
  ELF
+ ELF
 FOOL
```

Algebra.

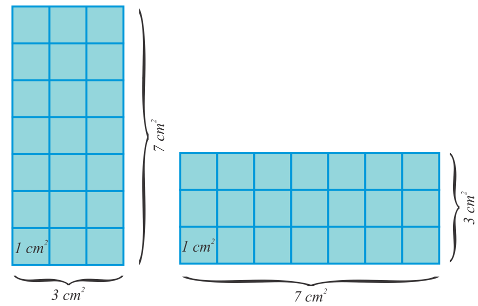
1. Natural numbers.

According to the modern studies, many animals' species have a sense of amount and quantity. In many different experiments animals have shown the ability to differentiate between smaller and bigger amount of food, quantities of things such as 1, 2, and 3. In captivity, after training, they can show even better results. Prehistoric people have introduced the special words to indicate the number of items in a group (number of elements in a set, as we are saying now). We can even assume that at the beginning, different words were used to specify the same number of the different objects. Only after thousands of years the abstract concept of "numbers" was separated from the number of real objects in a group. That moment can be considered as a beginning of mathematics.

2. Properties of the arithmetic operations.



Commutative and associative properties of addition are easy to understand. Multiplication is just a shorter way to write the addition of equal groups, so commutative and associative properties of multiplication can be visualized and understood with the help of the rectangle area. (See the piture). Areas of identical rectangles are equal,



$$S = 3cm^2 \cdot 7 = 7cm^2 \cdot 3 = 3cm \cdot 7cm = 21cm^2$$

The distributive property can be explained with the definition of multiplication as well;

$2 \cdot (3+7) = (3+7) + (3+7) = 3+3+7+7 = 2 \cdot 3 + 2 \cdot 7$ and it is true for any numbers.

We can do it he other way around:

$$2 \cdot 3 + 2 \cdot 7 = 3+3+7+7 = 3+7+3+7 = (3+7)+(3+7) = 2 \cdot (3+7)$$

The distributive property can be illustrated by the following problems:

Farmer put green and red grapes into boxes. Each box contains 5lb of grapes. How many pounds of green and red grapes altogether did farmer put into boxes if he had 10 boxes of green and 8 boxes of red grapes? Is there any difference between 2 following expressions?

$$5 \cdot (10+8) \text{ or } 5 \cdot 10 + 5 \cdot 8$$

What is represented by the first expression? By the second?

Another example:

For the party John bought 7 identical boxes of chocolates, 20 candies in each box. Guests ate 12 candies from each box. How many chocolates are left after the party?
Again, two numerical expression can be written to describe the problem:

$$7 \cdot (20-12) \text{ and } 7 \cdot 20 - 7 \cdot 12.$$

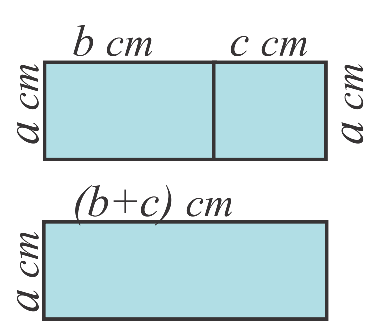
For both examples we can write the equality:

$7 \cdot (20-12) = 7 \cdot 20 - 7 \cdot 12$

$5 \cdot (10+8) = 5 \cdot 10 + 5 \cdot 8$

These equalities are numerical representation of the distributive property, which can be written in the general form as $a \cdot (b+c) = a \cdot b + a \cdot c$. (and of cause $a \cdot b + a \cdot c = a \cdot (b+c)$ is also true, this way of writing the distributive property is called the factoring the common factor out (of the parenthesis). The other way to see the distributive property is as an combined area of two rectangles with one side of the same length and the area of one rectangle. Combined area of two rectangles S_2 equals to $a \cdot b + a \cdot c$, and the area of one big rectangle is $S_1 = a(b+c)$:

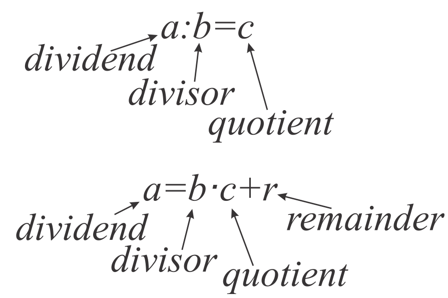
$$S_1 = a(b+c) = a \cdot b + a \cdot c = S_2$$



(see the picture on the right).

3. Divisibility.

We say that a natural number is divisible by another natural number if the result of this operation is a natural number. If this is not the case then we can divide a number with a remainder.



If a and n are natural numbers, the result of a division operation of $a \div n$ will be a quotient c, such that

$$a = b \times c + r$$

Where r is a remainder of a division $a \div b$. If r is 0, then we can tell that a is divisible by b.

- If we want to divide m by 15, what numbers we can get as a remainder?

If the remainder is 0, then quotient and divisor are both factors of dividend, $a = b \cdot c$, and to divide a number $\boldsymbol{a}$ by another number, $\boldsymbol{b}$, means to find such number $\boldsymbol{c}$, that $c \cdot b$ will give us a. So, because the product of 0 and any number is 0, than there is no such arithmetic operation as division by 0.

4. Divisibility rules.

A statement (or proposition) is a sentence that is either true or false, but not both. So '3 is an odd integer' is a statement.
But 'π is a cool number' is not a (mathematical) statement.
Note that '4 is an odd integer' is also a statement, but it is a false statement.
Are these sentences statements or not? If yes, are they true or false? Can you prove it?

- Telephone numbers in the USA have 10 digits.
- The moon is made of cheese.
- The sum of 2 odd natural number is an even number
- Would you like some cake?
- $3 + x = 12$
- The sum of two even numbers.
- $1 + 3 + 5 + 7 + \cdots + 2n + 1$.
- Go to your room!
- $7 + 3 = 10$
- All birds can fly.

The rule of divisibility by 2 is:
If the last digit of a number is an even number or 0 (0, 2, 4, 6, or 8) the number is even number (divisible by 2).

Divisibility Rules	
A number is divisible by	
2	If last digit is 0, 2, 4, 6, or 8
3	If the sum of the digits is divisible by 3
4	If the last two digits is divisible by 4
5	If the last digit is 0 or 5
6	If the number is divisible by 2 and 3
7	cross off last digit, double it and subtract. Repeat if you want. If new number is divisible by 7, the original number is divisible by 7
8	If last 3 digits is divisible by 8
9	If the sum of the digits is divisible by 9
10	If the last digit is 0
11	Subtract the last digit from the number formed by the remaining digits. If new number is divisible by 11, the original number is divisible by 11
12	If the number is divisible by 3 and 4

Proof of the divisibility by 2 rule:

Any natural number can be written as a sum:

$$\ldots + 1000 \cdot n + 100 \cdot m + 10 \cdot l + k = \cdots + 2 \cdot 500 \times n + 2 \cdot 50 \times m + 2 \cdot 5 \cdot l + k$$

Were n, m, l, and k are numbers of thousands, hundreds, tens, and units. If k is an even number or 0, it also can be represented as a product of 2 and another single digit number. Then the number can be written as:

$$\ldots + 1000 \times n + 100m + 10 \times l + k = \cdots + 2 \times 500 \times n + 2 \times 50 \times m + 2 \times 5 \times l + 2 \times p$$

(p can be 0, 1, 2, 3, and 4. Do you know why?). Distributive property is allowing us to represent this expression as a product:

$$\ldots + 1000 \times n + 100m + 10 \times l + k = \cdots + 2 \times 500 \times n + 2 \times 50 \times m + 2 \times 5 \times l + k$$
$$= 2 \times (\ldots + 500 \times n + 50 \times m + 5 \times l + p)$$

Now we can see that the number is divisible by 2 if its last digit is even or 0.

All other divisibility rules can be proved as well.

Factorization.

In mathematics factorization is a decomposition of one number into a product of two or more numbers, or representation of an expression as a product of 2 or more expressions, which called 'factors'. For example, we can represent the expression $a \cdot b + a \cdot c$ as a product of a and expression $(b + c)$. Can you explain why?

$$a \cdot b + a \cdot c = a \cdot (b + c)$$

Or in a numerical expression:

$$7 \cdot 5 + 7 \cdot 3 = 7 \cdot (5 + 3)$$

Or a number can be representing as product of two or more other numbers, for example:

$$40 = 4 \cdot 10 = 4 \cdot 2 \cdot 5, \quad 36 = 6 \cdot 6 = 3 \cdot 2 \cdot 6$$

Does any natural number can be represented as a product of 2 or more numbers besides 1 and itself? Natural numbers greater than 1 that has no positive divisors other than 1 and itself are called prime numbers.

Even numbers are the numbers divisible by 2 (they have 2 as a divisor), so they can be factorized as 2 times something else. Can an even number be a prime number? Is there any even prime number?

Prime factorization or integer **factorization** of a number is the determination of the set of **prime** numbers which multiply together to give the original integer. It is also known as **prime** decomposition.

168	*2*
84	*2*
42	*2*
21	*3*
7	*7*
1	

180	*2*
90	*2*
45	*3*
15	*3*
5	*5*
1	

Prime factorization process:

Prime factors of 168 are $2, 2, 2, 3, 7$ and prime factors of 180 are $2, 2, 3, 3, 5,$

$$2 \times 2 \times 2 \times 3 \times 7 = 168; \quad 2 \times 2 \times 3 \times 3 \times 5 = 180$$

Eratosthenes proposed a simple algorithm for finding prime numbers. This algorithm is known in

mathematics as the Sieve of Eratosthenes.

In mathematics, the sieve of Eratosthenes, one of a number of prime number sieves, is a simple, ancient algorithm for finding all prime numbers up to any given limit. It does so by iteratively marking as composite, *i.e.*, not prime, the multiples of each prime, starting with the multiples of 2.

~~1~~	2	3	~~4~~	5	~~6~~	7	~~8~~	~~9~~	~~10~~
11	~~12~~	13	~~14~~	~~15~~	~~16~~	17	~~18~~	19	~~20~~
~~21~~	~~22~~	23	~~24~~	~~25~~	~~26~~	~~27~~	~~28~~	29	~~30~~
31	~~32~~	~~33~~	~~34~~	~~35~~	~~36~~	37	~~38~~	~~39~~	~~40~~
41	~~42~~	43	~~44~~	~~45~~	~~46~~	47	~~48~~	~~49~~	~~50~~
~~51~~	~~52~~	53	~~54~~	~~55~~	~~56~~	~~57~~	~~58~~	59	~~60~~
61	~~62~~	~~63~~	~~64~~	~~65~~	~~66~~	67	~~68~~	~~69~~	~~70~~
71	~~72~~	73	~~74~~	~~75~~	~~76~~	~~77~~	~~78~~	79	~~80~~
~~81~~	~~82~~	83	~~84~~	~~85~~	~~86~~	~~87~~	~~88~~	89	~~90~~
~~91~~	~~92~~	~~93~~	~~94~~	~~95~~	~~96~~	97	~~98~~	~~99~~	~~100~~

Exercises:

1. Proof that the sum of two any even natural numbers is an even number.
2. The remainder of 1932 ÷ 17 is 11, the remainder of 261 ÷ 17 is 6. Is 2193 = 1932 + 261 divisible by 17? Can you tell without calculating? Explain.
3. Find all natural numbers such that upon division by 7 the quotient and remainder will be equal.
4. Even or odd number will be the sum and the product of
   a. 2 odd numbers
   b. 2 even numbers
   c. 1 even and 1 odd number
   d. 1 odd and 1 even number

   Can you explain why? (a few examples do not prove the statement).
5. Compute (what is the best way to compute it?):

a. $23 \times 15 + 15 \times 77;$

b. $79 \times 21 - 69 \times 21;$

c. $340 \times 7 + 16 \times 70;$

d. $250 \times 61 - 25 \times 390;$

e. $67 \times 58 + 33 \times 58;$

f. $55 \times 682 - 45 \times 682;$

6. Can the expression below be a true statement, if letters are replaced with numbers from 1 to 9 (different letters correspond to different numbers).

$$f \cdot l \cdot y = i \cdot n \cdot s \cdot e \cdot c \cdot t$$

Geometry.

A **definition** is a statement of the meaning of a something (term, word, another statement).

Desk *noun*

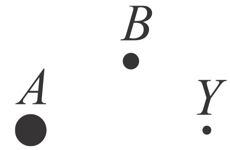
noun: **desk**; plural noun: **desks**

1. a piece of furniture with a flat or sloped surface and typically with drawers, at which one can read, write, or do other work.
   - Music
     a position in an orchestra at which two players share a music stand.
     "an extra desk of first and second violins"
   - a counter in a hotel, bank, or airport at which a customer may check in or obtain information.
     "the reception desk"

In mathematics everything (mmm,,,, almost everything) should be very well defined. In our real life, it is also very useful and convenient to agree about terms and concepts, to give them a definition, before starting using them just to be sure that everybody knows what they are talking about.   Now we move to geometry.

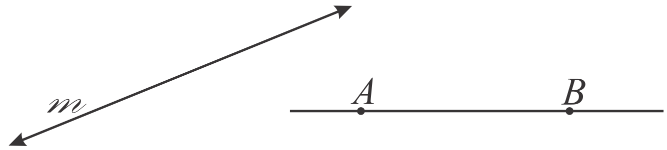
Can we give a definition to a point? Can we clearly define what a point is? What a line is? What a plane is? Mathematicians decided do not define terms "point", "straight line", and "plane" and to rely upon intuitive understanding of these terms.

Point (an undefined term).



In geometry, a point has no dimension (actual size), point is an exact location in space. Although we represent a point with a dot, the point has no length, width, or thickness. Our dot can be very tiny or very large and it still represents a point. A point is usually named with a capital letter.

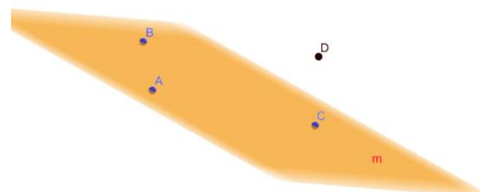
Line (an undefined term).

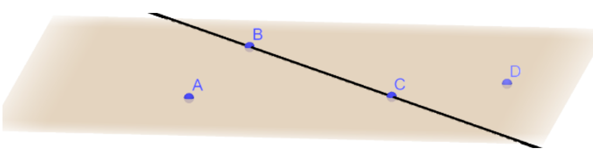


In geometry, a line has no thickness but its length extends in one dimension and goes on forever in both directions. Unless otherwise stated a line is drawn as a straight line with two arrowheads indicating that the line extends without end in both directions (or without them). A line is named by a single lowercase letter, *m* for example, or by any two points on the line, $\overleftrightarrow{AB}$ or AB.

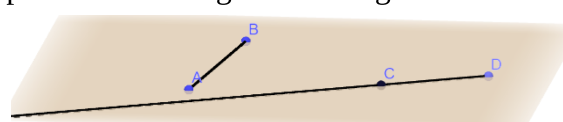
Plane (an undefined term).

In geometry, a plane has no thickness but extends indefinitely in all directions. Planes are usually represented by a shape that looks like a parallelogram. Even though the diagram of a plane has edges, you must remember that the plane has no boundaries. A plane is named by a single letter (plane *p*) or by three non-collinear points (plane ABC).





Points can belong to the plane or can be outside of the plane. On a plane, points can belong to the straight line, or can be positioned on ether half-plane.



A set of all points of a straight line between two specific points. These points are called endpoints.

A ray is a part of a straight line consisting of a point (endpoint) and all points of a straight line at one side of an endpoint. Ray is named by endpoint and any other point, ray $\overrightarrow{AB}$ or AB (where A is an endpoint)

Exercises:

1. Draw a segment 2 cm long, 5 cm long, a square with the side 4 cm. (use ruler, pencil).
2. Draw two segments AB and CD in such way that their intersect
   a. by a point
   b. by a segment
   c. don't intersect at all.

3. Using a ruler draw a straight line, put on it 3 points, *A*, *B*, and *C* so that 2 rays are formed, *BC* and *BA*.

4. Draw two rays AB and CD in such way that their intersect
   d. by a point
   e. by a segment
   f. by a ray
   g. don't intersect at all.

5. Through which points does the line *m* pass?
   Through which points does the line *a* pass?
   What is the intersection of the lines *m* and *l*?
6. Mark 2 points. How many different lines can be drawn through these two points?
7. Mark three points. How many lines can be drawn through three points?
   Consider all possible solution.
8. Mark four points. How many lines can be drawn through four points?