Homework 22.

Equivalence of mass and energy.

Let us return to our problem with inelastic collision of two balls.

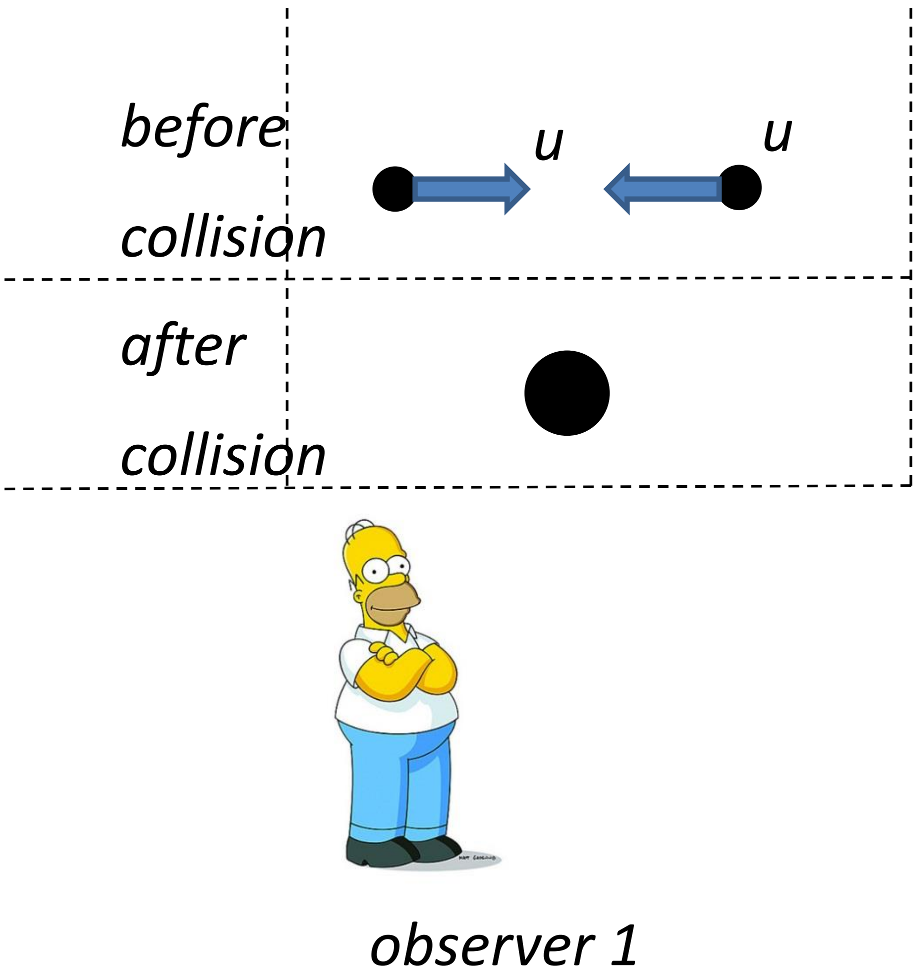

Figure 1.

Last time we learned that total energy of a particle can be expressed as:

$$E = mc^2 = \frac{m_0 c^2}{\sqrt{1 - \frac{v^2}{c^2}}} \qquad (1),$$

where m is relativistic mass. Total energy of a particle can be represented as a sum of to parts, rest energy and kinetic energy ***K***:

$$E = m_0 c^2 + K \quad (2).$$

This was, again, the expressions for a single particle. How can we express the total energy of a system of interacting particles? When discussing inelastic collision of two balls we demonstrated that if we assume that the relativistic mass conserves, than the total energy E of the two balls conserves as well. But if we take a look at equation (2) we will see that if the total energy E conserves than any change in kinetic energy K has to be counterbalanced by the corresponding change in the rest mass (the speed of light is a constant). So we can assume that for the system of interacting particles any change in total kinetic energy of the particles will also be compensated by equal but opposite change of their total rest energy.

For observer 1 in Figure 1the big ball after collision is at rest, so we can write:

$$M_0 = 2m = \frac{2m_0}{\sqrt{1-\frac{v^2}{c^2}}} \quad (3).$$

We can see that although relativistic mass is conserved in the collision, the rest mass is not. The rest mass is increased and the increase of the rest mass Δm_0 is:

$$\Delta m_0 = M_0 - 2m_0 = 2m_0\left(\frac{1}{\sqrt{1-\frac{v^2}{c^2}}} - 1\right) \quad (4).$$

Let us calculate the change in kinetic energy ΔK. Before collision the total kinetic energy is:

$$K_{before} = 2(E - m_0 c^2) = 2m_0\left(\frac{1}{\sqrt{1-\frac{v^2}{c^2}}} - 1\right)$$

After the collision no kinetic energy remains, $K_{after}=0$. So, the change in kinetic energy is :

$$\Delta K = K_{after} - K_{before} = -2m_0\left(\frac{1}{\sqrt{1-\frac{v^2}{c^2}}} - 1\right)c^2$$

$$\Delta K = -\Delta m_0 c^2$$

We can see that the decrease of kinetic energy is counterbalanced by increase in rest energy, which ,in turn, is the increase of the rest mass multiplied by c^2. It turns out that the equivalence of mass and energy is a general principle which can be extrapolated far beyond the simple problem we have discussed. “Equivalence” means that the mass and energy are two “sides” of a single physical quantity which can be called *mass-energy*. So two separate conservation laws, conservation of mass and energy, which we know from classical mechanics merge into a single law of conservation of mass-energy in relativistic mechanics. Any mass can be expressed in energy units by simple multiplication by c^2.

The rest-mass energy of a body can be considered as the internal energy of the body in contrast to the kinetic energy, which can be defined as the “external” one. If we have, say a liter of water, than all the rest mass energies of the electrons and nucleus of the hydrogen and oxygen atoms, all the kinetic energies of molecules, all the electrical potential energies of interaction between electrons and nucleus, etc. etc., are in the rest energy of the water, and , hence in the total rest mass of the liter of water which is ~1kg at 0^0C. If we heat up the water we will change its internal energy, and, hence, its rest mass.

Problem:

1. A deuteron, the nucleus of heavy hydrogen, consists of a proton and neutron. A rest mass of a deuteron is 2.01355u, where u stands for atomic mass unit (1.66×10^{-27}kg). The rest mass of a proton is 1.00728u, the rest energy of a neutron is 1.00867u. Calculate the energy which is necessary to separate a deuteron to a proton and a neutron.

2. The Earth receives radiant energy from the Sun at the rate of 1340W/m^2. How long does it take for the Sun to lose all its mass due to radiation? The Sun’s mass is about 2×10^{30}kg.