Homework 17

Length contraction and time dilation.

We have learnt that the classical (they are also called "Galilean") formulae for connecting the coordinates in a reference frame where the observer is at rest (x,y,x,t) with the coordinates in a reference frame moving at a velocity V along x-axis (x_1,y_1,z_1, t_1) are:

$$\begin{aligned} x_1 &= x - Vt \\ y_1 &= y \\ z_1 &= z \\ t_1 &= t \end{aligned} \qquad (1)$$

They are approximate – they work well as long as velocity V is much less than the speed of light in vacuum c. The exact transformations (Lorentz transformations) are given below:

$$\begin{aligned} x_1 &= \frac{x - Vt}{\sqrt{1 - \frac{V^2}{c^2}}} \\ y_1 &= y \\ z_1 &= z \\ t_1 &= \frac{t - \frac{Vx}{c^2}}{\sqrt{1 - \frac{V^2}{c^2}}} \end{aligned} \qquad (2)$$

The most interesting feature in the formulae (2) is that now the time "flow" depends on the velocity of the reference of frame. In a moving reference of frame the time "slowed down". If a cock moves with a velocity V with respect to an observer its rate is measured to have slowed down by a factor $\sqrt{1 - V^2/c^2}$

Let us consider a time interval $t_2' - t_1'$, measured using the clock which are at rest in a reference frame moving along x-axis at a velocity V with respect to the observer's reference frame. From the Lorentz transformations (2), for the time interval $t_2 - t_1$in the observer's reference frame we have:

$$t_2 - t_1 = \frac{(t_2' - t_1') + (x_2' - x_1')\frac{V}{c^2}}{\sqrt{1 - \frac{V^2}{c^2}}} \qquad (3)$$

But the clock are at rest in the moving reference frame, so $(x_2' - x_1') = 0$. So, we have:

$$t_2 - t_1 = \frac{t_2' - t_1'}{\sqrt{1 - \frac{V^2}{c^2}}}, or\ \Delta t = \frac{\Delta t'}{\sqrt{1 - \frac{V^2}{c^2}}} \qquad (4)$$

You may think that now it is possible to say who is moving and who is staying – just check whose watch is slower. But, it does not work: your classmate will be convinced that it is *your* stopwatch which is slow. For him or her time is going normally. It is still not possible to determine who is moving and who is staying. So even if you are traveling in space at a

very high speed and, for those who are left on the Earth your time is much slower than the Earth time, you will not enjoy extended life: in your reference frame time is "normal" as well as aging, unfortunately.
Same is for the length: the length of a moving object is reduced in the direction of motion (length contraction):

$$x_2 - x_1 = (x_2' - x_1')\sqrt{1 - \frac{V^2}{c^2}}, or\ L = L_0\sqrt{1 - \frac{V^2}{c^2}} \qquad (5)$$

Here we assumed that the extended object is at rest in a moving reference frame: $L_0 = x_2' - x_1'$, where L_0 is the length of the object along x-axis. And again, this effect is only "seen from outside" – from a reference frame which moves with respect to yours at a speed V.

Problems:

In the problems below ***c*** is the speed of light in vacuum.

1. Prove formula (5).

2. Find perimeter of a square moving at a velocity V along one of its sides.

3. How do you "see" a density of a led cube if you are moving along one of the edges of the cube at a velocity c/2?

4. Find the time dilation for a clock moving at a velocity V=240000 km/s.