Homework 16

Lorentz transformations.

The result of Michelson-Morley experiment became clear after the work published by A. Einstein in 1905. In this work he demonstrated the constancy of the speed of light in all inertial reference frames.

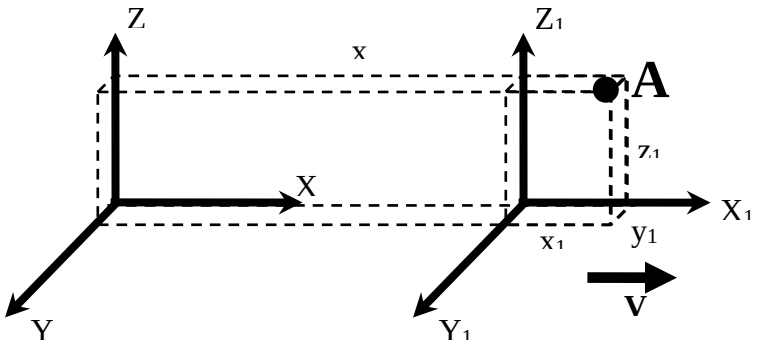

Fig.1

Let us consider a coordinate system $X_1Y_1Z_1$ which is moving at a velocity ***V*** with respect to the XYZ frame along the x axis (Fig.1). Let us imagine that the object A has coordinates x,y,z in the XYZ reference frame. We are going to express the coordinates x_1, y_1, z_1 of A in $X_1Y_1Z_1$ reference frame. According to classical nonrelativistic mechanics the transformation rules are:

$$\begin{aligned} x_1 &= x - Vt \\ y_1 &= y \\ z_1 &= z \\ t_1 &= t \end{aligned} \qquad (1)$$

The last equation looks absolutely trivial and intuitive. It means that the time flow is the same in both reference frames. If two events happen at the same time in one reference frame, they are expected to happen simultaneously in any other reference frame. A bit later, we will see that this is not true.

Michelson-Morley experiment suggested that the speed of light is the same in all inertial reference frames. This is in contradiction with the simple velocity composition rule which follows from the transformation rules (1) which are called Galilean transformations. Looks like the expressions (1) has to be corrected. Albert Einstein suggested the corrections. In a nice book "Basic concepts in relativity and early quantum theory" by R. Resnick and D. Halliday we read:

"In a paper entitled "Conversations with Albert Einstein" R. S. Shankland writes: "I asked Professor Einstein how long he had worked on the Special Theory of Relativity before 1905. He told me that he had started at age 16 and worked for ten years; first as a student when, of course, he could spend only part-time on it, but the problem was always with him. He abandoned many fruitless attempts, 'until al last it came to me that time was suspect'" What was it about time that Einstein questioned? It was the assumption, often made unconsciously and certainly not stressed that there exist a universal time that is the same for all observers<…> In prerelativistic discussions, the assumptions was there implicitly by the absence of a transformation equation for t in the Galilean equations."

Albert Einstein
(1879-1955).

In class we discussed the way to obtain the correct expressions. First, we assumed that space and time are homogeneous. This means that all pints in space and moments of time are equivalent. It follows from the homogeneity assumption that the result of a measurement of a length or a time interval between two specific events should not depend on where or when the interval happens to be in our reference frame. Second, we used postulates of the special theory of relativity:

1. All the inertial reference frames are equivalent
2. The speed of light in vacuum is the same in all the reference frames.

Then, the correct transformations are:

$$x_1 = \frac{x - Vt}{\sqrt{1 - \frac{V^2}{c^2}}}$$
$$y_1 = y$$
$$z_1 = z \qquad (2)$$
$$t_1 = \frac{t - \frac{Vx}{c^2}}{\sqrt{1 - \frac{V^2}{c^2}}}$$

Here, c is the light speed in vacuum. These formulae – Lorentz transformations - were originally suggested by Hendrick Lorentz (1899) and, independently, by Joseph Larmor (1897). The most interesting feature in the formulae (2) is that now the time "flow" now depends on the velocity of the reference of frame.

As long as the velocity of the moving reference frame is equal to the speed of light the denominators of the first and last transformations in (2) become zero – this means that the Lorentz transformation do not make sense

Problems:

In the problems below ***c*** is the speed of light in vacuum.

1. Imagine that an astronaut is moving at a speed 0.8c relative to the Earth. Find how long is his (her) hour as it is "seen" from the Earth?

2. Two events happen at the same time in a certain reference frame. Do you think that these events are simultaneous in all inertial reference frames? Proof your answer.

3. In a certain reference frame two events happen at different moments of time in different places. Is it always possible to find another inertial reference frame in which these events will happen at the same place?