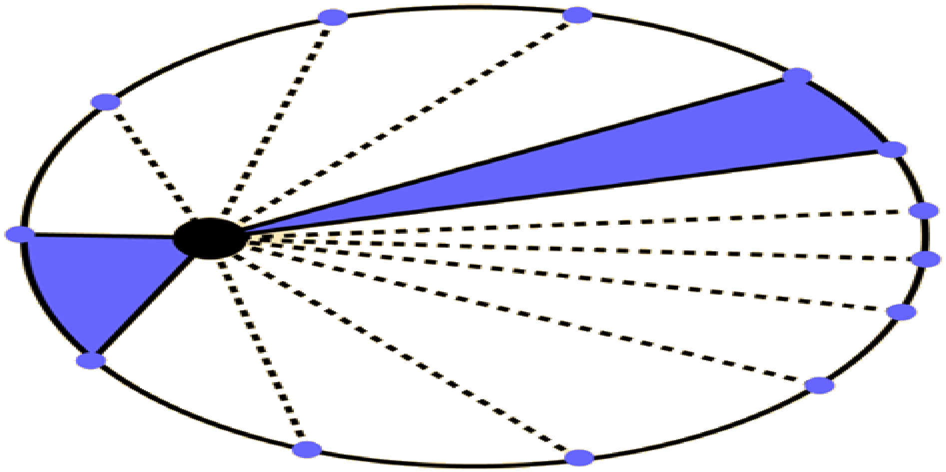
Kepler's Laws

I. The Law of Orbits: All planets move in elliptical orbits, with the sun at one focus.

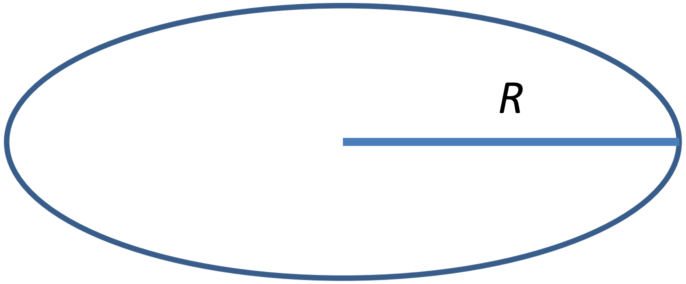
II. The Law of Areas: A line that connects a planet to the sun sweeps out equal areas in equal times.

$$\frac{\Delta A}{\Delta t} = const$$



III. The Law of Periods: The square of the period of any planet is proportional to the cube of the semimajor axis of its orbit (i.e. the "bigger" radius of the ellipse):

$$T^2 = const \times R^3$$



Newton's Law of Gravity

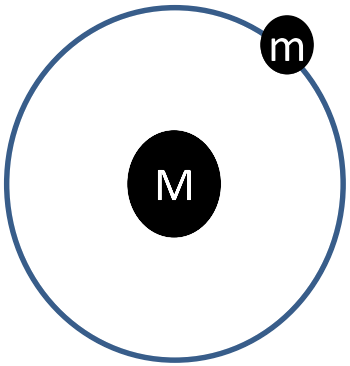
Two masses, m_1 and m_2 , experience *gravitational attractive force* to each other, that depends on distance between them, r:

$$F = -\frac{Gm_1m_2}{r^2}; \qquad G = 6.7\times10^{-11}\frac{m^3}{kg\cdot s^2}$$

G is called Gravitational Constant.
From Newton's Universal Gravity, one can derive Kepler's Laws. The easiest is to derive the Third Kepler's Law for the case of a circular orbit.

Consider a planet of mass ***m*** on a circular orbit of radius ***R*** around a star of mass ***M***. Since its centripetal acceleration, $\boldsymbol{a=v^2/R}$, is due to gravity, we obtain:

$$\frac{GMm}{R^2} = ma = m\frac{v^2}{R}$$



Here, speed $v = 2\pi R/T$, and therefore, Kepler was right!

$$T^2 = \frac{4\pi^2}{GM}R^3$$

Homework

Problem 1. By using a powerful telescope, you discovered a new planet and a satellite that orbits it very close to its surface (i.e. radius of the orbit is almost equal to the radius of the planet). The satellite make one rotation in 2 hours. Can you find the average density of the planet?

Problem 2. If we were living in a 2D world, the Newton's Gravity would have a slightly different form:

$$F = -\frac{G'Mm}{R}$$

Can you predict how would Kepler's Third Law change? Assume a circular orbit