# Work and Kinetic Energy

$2^{nd}$ Newton's Law can be rewritten as:

$$m\Delta v = F\Delta t$$

$$mv\Delta v = F(v\Delta t)$$

One can show that the left-hand-side is a change in Kinetic energy, $K = \frac{mv^2}{2}$

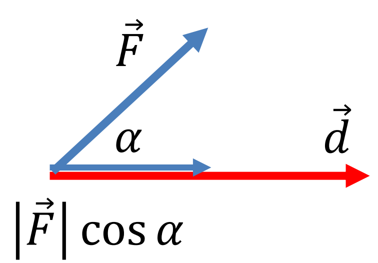
The right hand side (Force times Displacement) is called work, $W = F\Delta x$

This leads to a very important result:

$$\Delta K = W$$

A more general definition of work when Force and Displacement are given as vectors $\vec{F}$ and $\vec{d}$ (in 2D or 3D case):

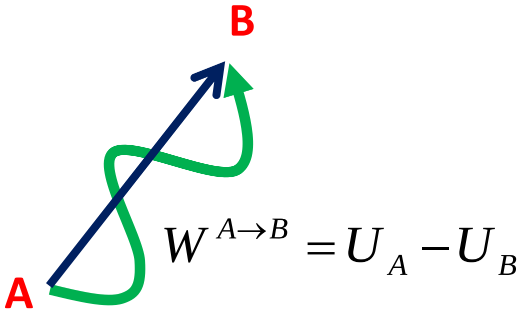
$$W = |\vec{F}||\vec{d}|\cos\alpha$$



# Conservation of Energy

Work by forces such as gravity of electrostatics depend only on initial and finite point, but not on the path itself. These forces are called Conservative (or Potential). For them we can introduce Potential energy, U:

$$W_{conservative}^{A\to B} = U_A - U_B = -\Delta U$$




$$\Delta K = -\Delta U + W_{non-conservative}$$

Therefore, if there is no non-conservative forces (no friction, engine or other external force), Total Energy (Kinetic + Potential) is conserved:

$$E = K + U = const$$

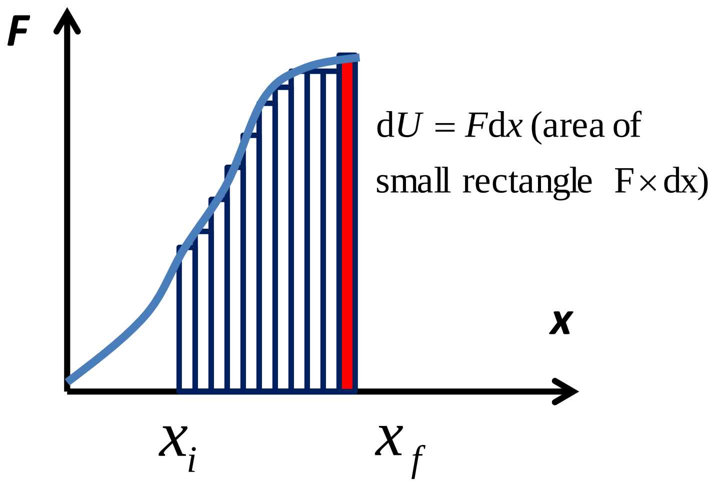
More generally, its change is equal to work of non-conservative forces:

$$\Delta E = W_{non-conservative}$$

# Potential Energy = Integral of Force

Let us  calculate the work done ***against*** the a force.
This work is stored as potential energy:

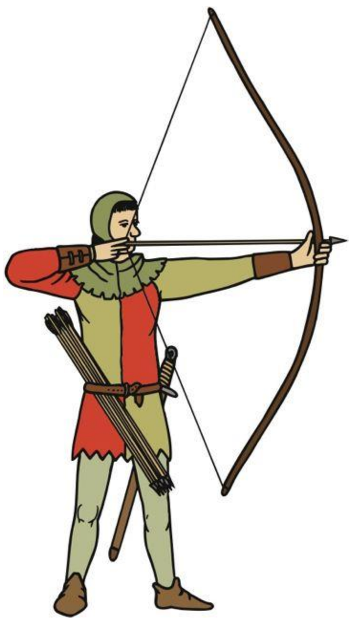
$$\Delta U = W = \int_{x_i}^{x_f} F(x)\mathrm{d}x$$




| Type of force | F | U |
|---|---|---|
| Gravity (on Earth surface) | mg | mgh |
| Hooke's Law (spring force) | kx | $\frac{kx^2}{2}$ |
| Newton's Law of Gravity | $F = \frac{Gm_1m_2}{r^2}$ | $U = -\frac{Gm_1m_2}{r}$ |

# Homework

**Problem 1**



English Longbow was an extremely powerful weapon that gave England big advantage in Middle Ages. An archer had to apply the force of approximately 500 N (equivalent of 50kg) to pull the string back by approximately 70 cm.

a) How much energy was carried by a single shout? Compare this to the energy of a 8 gram bullet shout by Kalashnikov machine gun with speed 700 m/s. Model the Longbow as a simple spring (what would be its spring constant?).

b) What is the maximum height that the arrow of mass m=50g could reach after being shut with the Longbow (assume no air drag)?

**Problem 2**

A pendulum is made of a ball attached to a weightless string of length *l*. The pendulum has been deflected by angle $\alpha$ from its stable vertical orientation (see the Figure). Find the maximum speed of the ball after the pendulum is released. Neglect air resistance.