Velocity and Acceleration (contd)

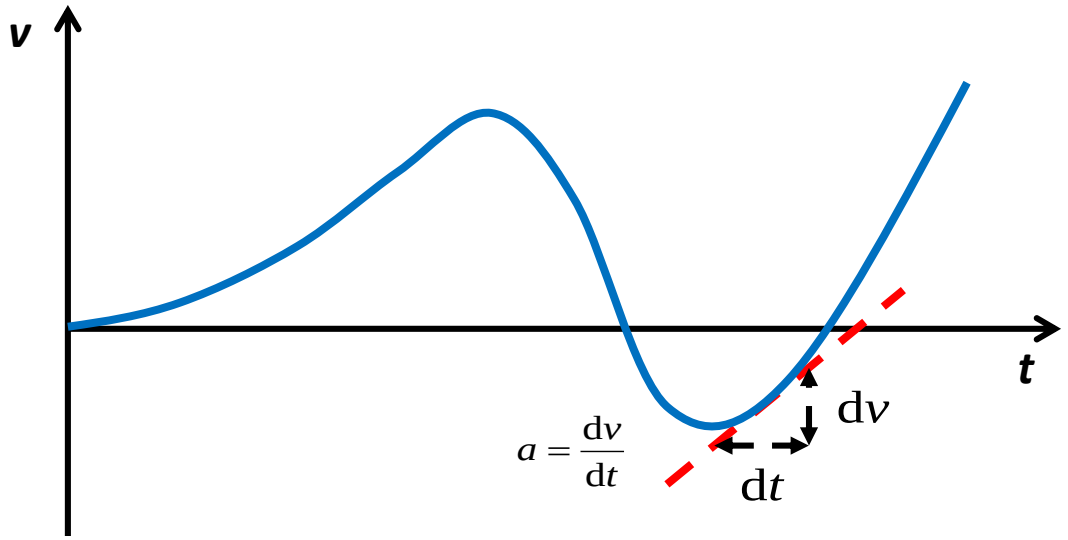
Last time, we defined **velocity** as a time derivative of a **position**:

$$v \underset{\Delta t \to 0}{=} \frac{\Delta x}{\Delta t} = \frac{\mathrm{d}x}{\mathrm{d}t}$$

Similarly, **acceleration** is the time derivative of **velocity**.

$$a = \frac{\mathrm{d}v}{\mathrm{d}t}$$

In other words, it is the rate of change of velocity, or local slope of the plot v(t) :



In 2D and 3D, one has to find a derivative of each of the vector components independently. For instance:

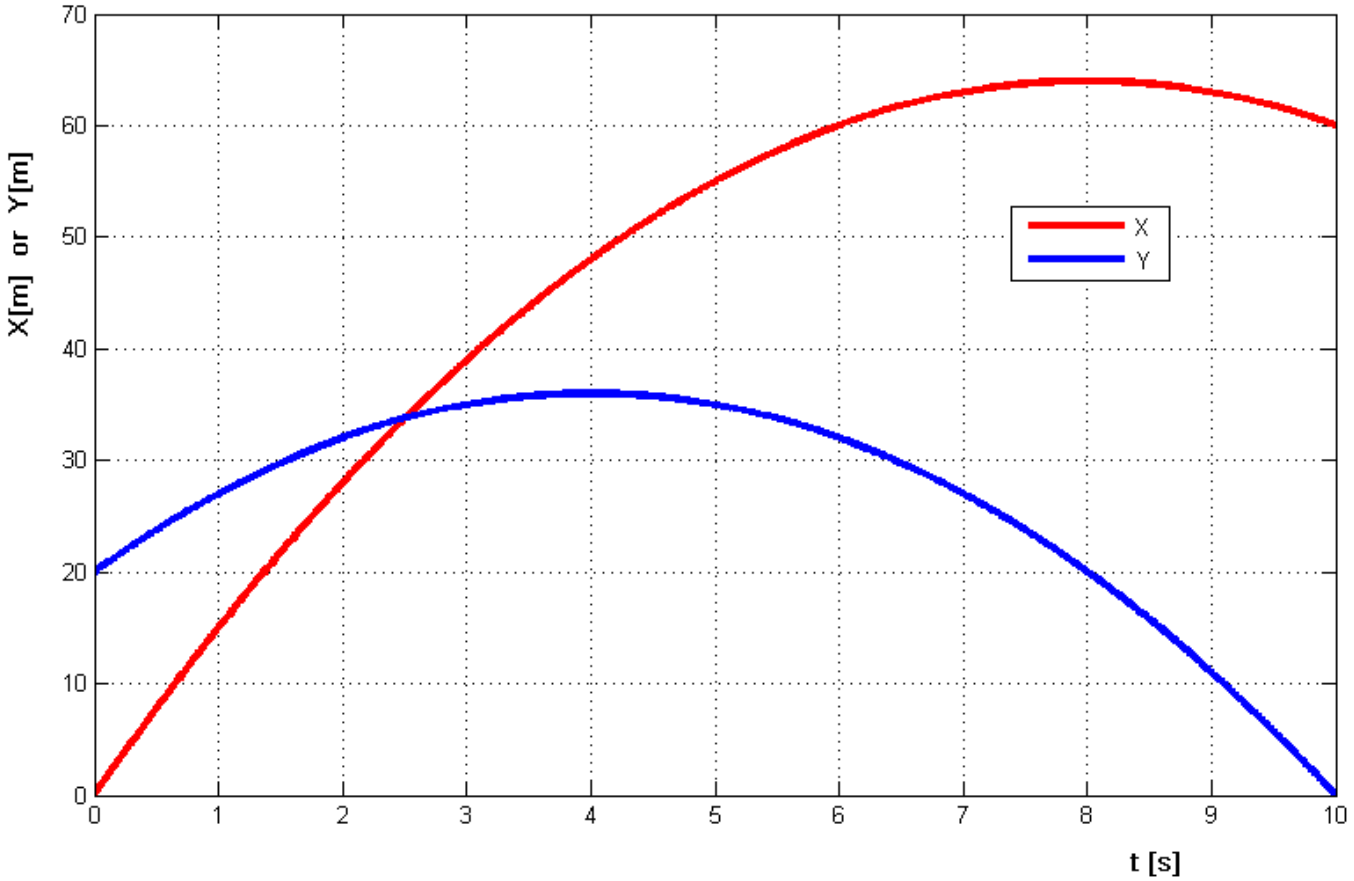
$$\vec{v}_{3D} = \left(\frac{\mathrm{d}x}{\mathrm{d}t}, \frac{\mathrm{d}y}{\mathrm{d}t}, \frac{\mathrm{d}z}{\mathrm{d}t} \right), \quad \text{magnitude (length) of this vector is instantaneous speed.}$$

Problem 1.

The two curves on the plot below show the two coordinates (x and y) of a Jet Ski , as functions of time t. Use this information to

a) Find the velocity vector $\vec{v} = (v_x, v_y)$, and speed v at time t= 6s.

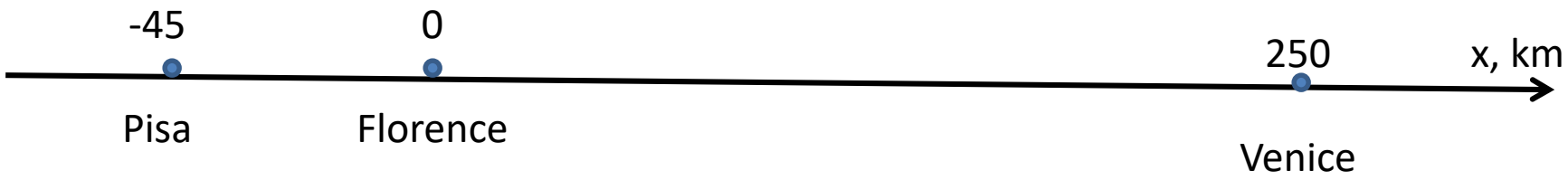
b) sketch the overall trajectory of the Jet Ski in (x,y) coordinates.



Problem 2.

While on vacation in Pisa, Italy, James Bond learns that a villain named Dr. Nope from Venice is about to purchase a secret and deadly weapon code-named Big Mac. It is to be sold by certain Merchant based in the city of Florence. Florence is on the way between Pisa and Venice, distance from Florence to Pisa is d_1=45 km, and from Florence to Venice is d_2=250 km. Both the Merchant and Dr. Nope want to meet and finish the deal as soon as possible.

As a result, all three of them get to their cars. Dr. Nope starts from Venice and drives towards Florence with average speed v_1=140 km/hr. At the very same moment, the Merchant (from Florence) and James Bond (form Pisa) start driving towards Venice. The Merchant's speed is v_2 = 160 km/hr.



a) How much time will it take for Dr. Nope and the Merchant to meet, if Bond is detained by Italian Police for speeding? Try to get the formula for this time t, before computing the number.

b) In fact, Mr. Bond did escape from the police chase. What must be Bond's average speed to ensure that all three of them meet at the same point (to have prolonged fight with shooting and special effects)? Again, try to derive the formula first.

c) Plot the coordinate x of all three characters versus time, t.