Homework 25.

Radioactivity.

We started discussing radioactivity. Radioactivity is a process of transformation of an atomic nuclei accompanied by emission of high energy particles. We learned that there are three major kinds of particles which can be emitted as a result of radioactive decay. These are α-, β- and γ-particles.

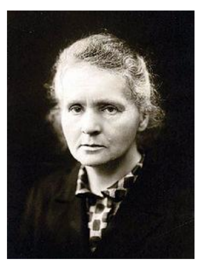
α-particle is a heavy, positively charged particle, which consists of 2 protons and 2 neutrons. After emission of a α-particle the atomic number of an element decreases by 2 and the mass number decreases by 4.

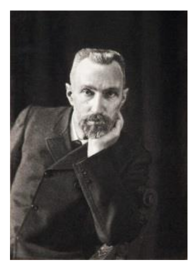
A neutron is not a stable particle and can turn into a proton. This process is called β^--decay and is accompanied by emission of an electron (so the total charge is conserved) and another particle which is called electron antineutrino. (β^+-decay with the emission of positron (anti-electron) is also possible, but we will not discuss it now). At β^--decay the atomic number increases by 1, but the mass number does not change.

A γ-particle is a high energy particle of electromagnetic radiation. This particle has no electric charge.

The elements whose atoms experience radioactive decay are called *radioactive*, the other elements are called *stable*.

The discovery of radioactivity is associated with three names:





Henri Becquerel (1852-1908) Marie Skłodowska-Curie (1867-1934) Pierre Curie (1859-1906)

We learned that each radioactive element is characterized by a decay constant and a half-life. Mathematically, the radioactive decay can be characterized by the following way. Let us assume that initially we have N_0 atoms of a radioactive element, say, uranium, U. As the time goes, the atoms will experience radioactive decay and will be transforming to atoms of another element. The number of Uranium atoms will be decreasing according to the expression:

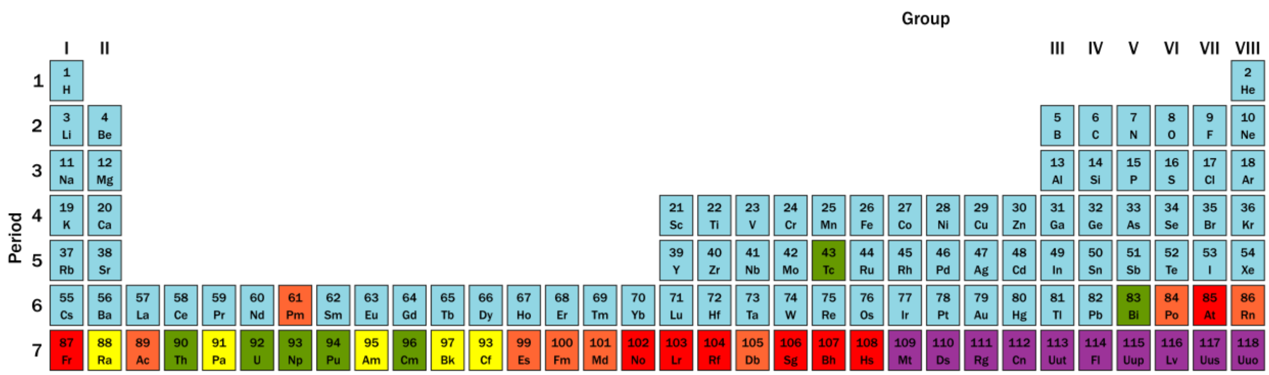
$$N(t) = N_0 \cdot e^{-\frac{t}{T}} \qquad (1)$$

Such a function is called “exponential function”. Let us consider formula 1, which looks confusing for a first glance.

a) N(t) the number of uranium atoms in time t,
b) ***e*** is a special number **e**=2.718281828459…It is called the Euler’s number or Napier’s constant. Similar to the famous π, it has infinite string of digits after the point and this string does not have a period. In physics the Euler number is as important as π. If you take a large number n, say n=1,000,000 and calculate$(1+1/n)^n$ the result will be close to **e**. For 1,000,000 I got 2.7182804693… .The larger is n, the closer to **e** is the result. For our calculations we can round e to the second sign and use 2.72.
c) What does the negative power mean? For example, 2^2=4, but $2^{-2} = \frac{1}{2^2} = \frac{1}{4}$
d) T is the decay time. Each radioactive element has its own decay time. After time T, the initial number of radioactive atoms decreases e times. How do we know that? Let us use the formula (1) and plug T instead of t. We get:

$$N(T) = N_0 e^{-\frac{T}{T}} = N_0 e^{-1} = \frac{N_0}{e} \qquad (2)$$

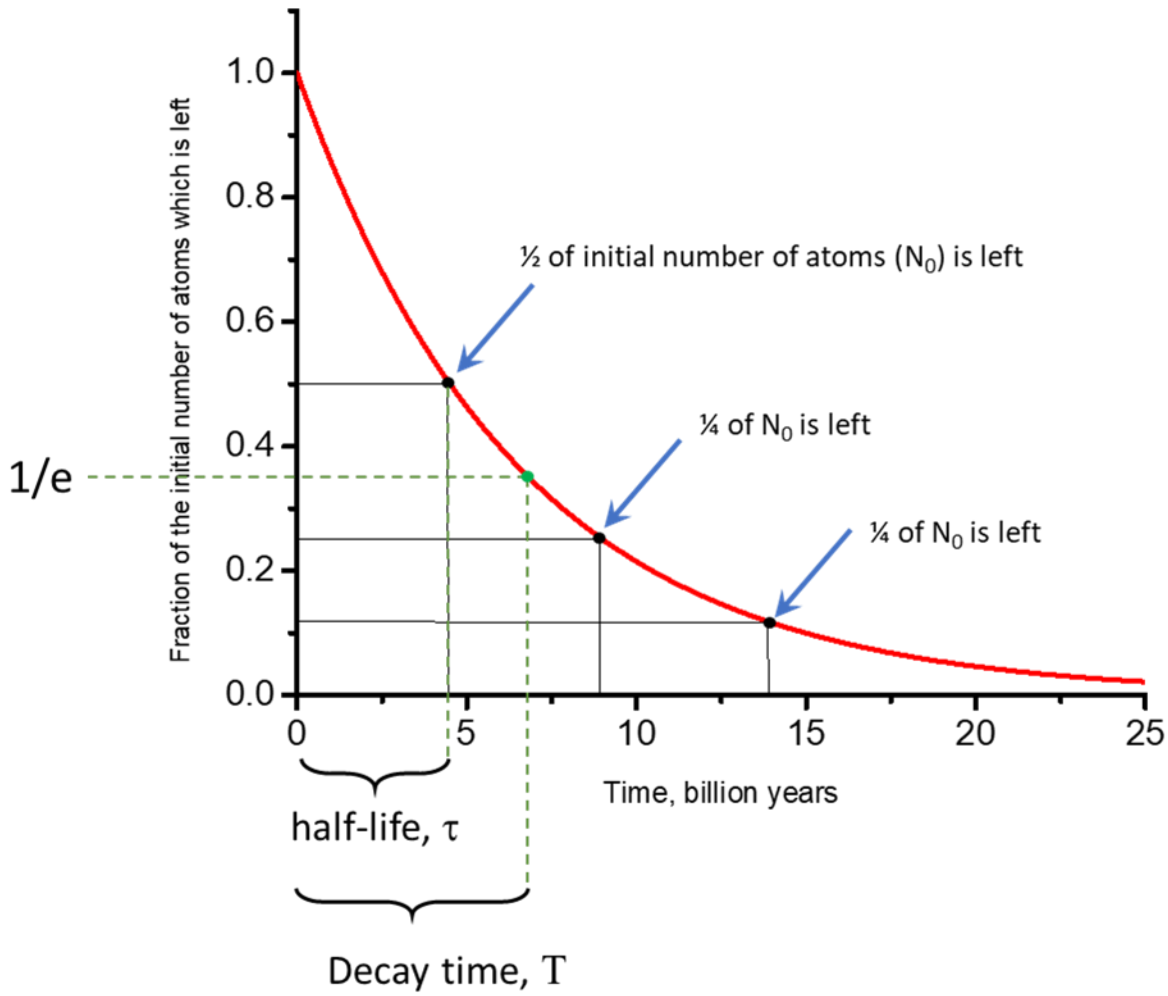
Radioactive elements and their time constants are shown in Figure below:



Periodic table with elements colored according to the half-life of their most stable isotope. 32-column format is helpful in showing stability pattern. Elements that contain at least one stable isotope. Radioactive elements: the most stable isotope is very long-lived, with half-life of over four million years. Radioactive elements: the most stable isotope has half-life between 800 and 34.000 years. Radioactive elements: the most stable isotope has half-life between one day and 130 years. Highly radioactive elements: the most stable isotope has half-life between several minutes and one day. Extremely radioactive elements: the most stable isotope has half-life less than several minutes. Very little is known about these elements due to their extreme instability and radioactivity. Note that this table only addresses the most stable isotope of each element. Therefore it would be erroneous to conclude that all naturally occurring elements from hydrogen through lead are non-radioactive. For example, if you hold a geiger counter up to a banana, you will detect radioactivity because of the potassium isotope ^{40}K, which is also the most common radioisotope in the human body.

Picture and caption are taken from: https://commons.wikimedia.org/wiki/User:Morgan_Phoenix

Let us make a plot of the formula 1 for uranium-238 (238 is the mass number) with the decay time of ~ 6.5 billion years.



From the graph we can see that in ~4.5 billion years ½ of the initial number of atoms will be left. I we will wait for another 4.5 billion years, this "half" will again decrease two times, so we will have ¼ of the initial amount. In 4,5x3=13.5 billion years only 1/8 of the initial amount will survive. This is one of the magic properties of the exponential function: if we increase (decrease) x by a fixed amount (let us call it *increment*), say 0, 3,6,9, (here the increment is 3)…the corresponding values f(x) increase (decrease) a certain number of times. Depending on the increment it may be 2 times or 3 times or e times. Each time we add the decay time T to the time t, the initial number of atoms decreases e times.

Sometimes, instead the decay time it is more convenient to use half-life **τ**. Half-life is the time period after which only half of the original number of atoms is left. The decay time is related to the half-life:

$$\tau \approx T \cdot 0.69$$

After the decay the atoms do not disappear – they transform to atoms of another chemical element depending of the number of protons in the nucleus after the decay happened.

Problem:

Initially you have 420g of Po^{210}. The half-life of Po^{210} is 138 days. How many atoms will decay during 30 day period? Assuming that the type of the decay is α-radioactivity, what will be the resulting element?