

Homework 17.

Last class we discussed parallel and series connection of capacitors. We learned that for parallel connection the resulting capacitance is the sum of the capacitances:

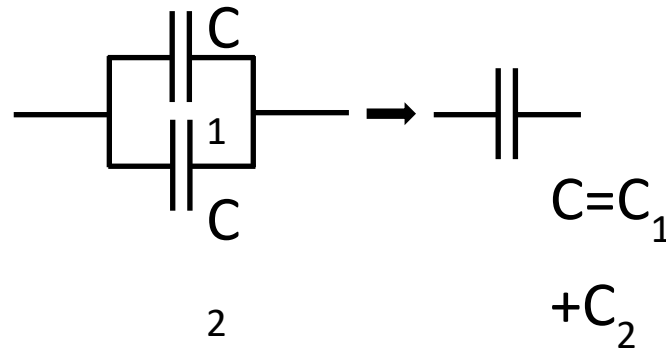


Figure 1. Parallel connection of the capacitors

Since the total voltage U on two elements connected in parallel is the same ($U_{\text{total}} = U_1 = U_2$), but the total area of the capacitor plates is now the sum of the plate areas of the capacitors, we can expect that total charge on the “negative” (or “positive”) plates is also the sum of the charges on the corresponding plates of two capacitors: $Q_{\text{total}} = Q_1 + Q_2$. So, in comparison with just one capacitor, we need more charge for the same voltage. The total capacitance is the ratio of the total charge to the total voltage:

$$C = \frac{Q_{\text{total}}}{U_{\text{total}}} = \frac{Q_1 + Q_2}{U_{\text{total}}} = \frac{Q_1}{U_{\text{total}}} + \frac{Q_2}{U_{\text{total}}} = C_1 + C_2 \quad (1)$$

For parallel connection the resulting capacitance can be found as shown below:

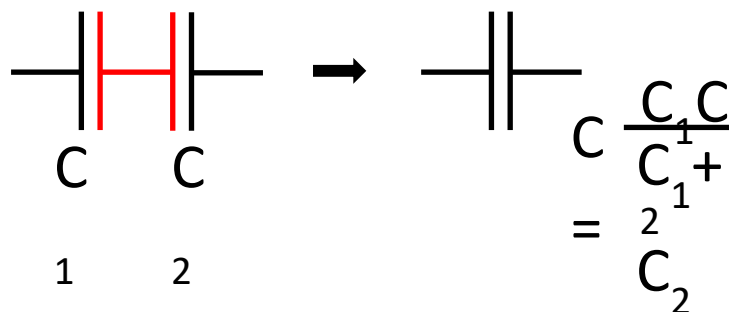


Figure 2. Series connection of the capacitors

In this case the charge induced on the corresponding plates of both capacitors is the same. We can understand that by considering the part of the circuit shown in red. This part includes one plate of the capacitor C_1 which is connected to a plate of the capacitor C_2 . After we connect the capacitors to a voltage source, the total charge of this part will remain zero, since it is disconnected from the rest of the circuit and it has had no charge before we connected the battery. So the charges on “inner” (red, Figure 2) plates on the capacitors C_1 and C_2 are equal in magnitude but have opposite signs. The charges on the “outer” plates of the capacitors are equal to those on the corresponding “inner” plates: $Q_{\text{total}}=Q_1=Q_2$. The total voltage on two capacitors U connected in series is the sum of the voltages on each capacitor: $U_{\text{total}}=U_1+U_2$. We can express the total voltages on the capacitors as

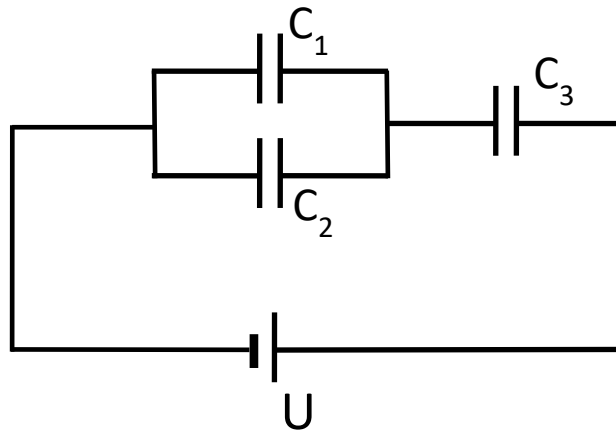
$$U_{\text{total}} = U_1 + U_2 = \frac{Q_{\text{total}}}{C_1} + \frac{Q_{\text{total}}}{C_2} = Q_{\text{total}} \left(\frac{1}{C_1} + \frac{1}{C_2} \right) = \frac{Q_{\text{total}}}{C}, \quad (2)$$

where

$$\frac{1}{C} = \frac{1}{C_1} + \frac{1}{C_2}, \quad \text{or} \quad C = \frac{C_1 C_2}{C_1 + C_2} \quad (3)$$

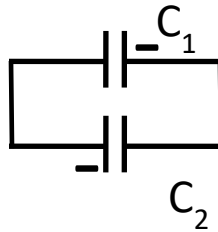
Problems:

1. Find the charge on each capacitor at the circuit below:



Assume that you know capacitance of each capacitor and the voltage of the battery (U).
(remember that if two circuit elements are connected in parallel then their voltages are equal; if two elements are connected in series then the total voltage is the sum of the individual voltages)

2. Two capacitors have charges q_1 and q_2 and capacitances C_1 and C_2 . We are connecting the plates of one capacitor to the plates of the other as shown in the Figure below:



Find the charges on the capacitors after the connection.