Homework 6.

Superposition.

The beauty and convenience of the concept of electric potential is that using the electrical potential we can easily calculate the potential energy of a charged object in the electric field created by arbitrary configuration of other charged objects.

Let us consider the following example.

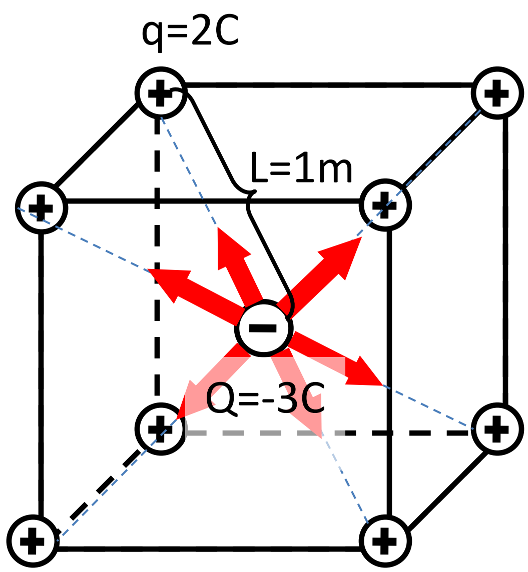

Figure 1. The red arrows show the electrostatic forces applied to the negative charge.

Given:

Eight point charges q=2C each are placed in the corners (vertices) of a cube (see Figure 1 above). The positions of the positive charges are fixed. The distance between the center and a corner of the cube is 1m. A negative charge of Q=3C is placed in the center of the cube.

Find electrostatic potential energy of the negative charge.

Solution:

As we remember, a possible way to calculate the electrostatic potential energy of a charge in a certain point is to calculate the electrostatic potential in this point and multiply it by the charge. Let us calculate the electrostatic potential in the center of the cube (black point in the Figure 2.

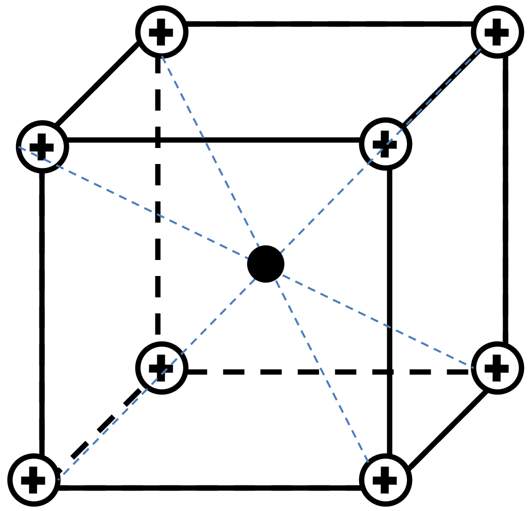

Figure 2.

At a first glance the problem looks difficult. But, in fact, it is not. To solve it, we will use the ***principle of superposition.*** According to this principle, we can calculate the potentials created in the center of the cube by each of the positive charges separately. After that we will just add these potentials together.

a) Let us pick just one positive charge (Figure 3).

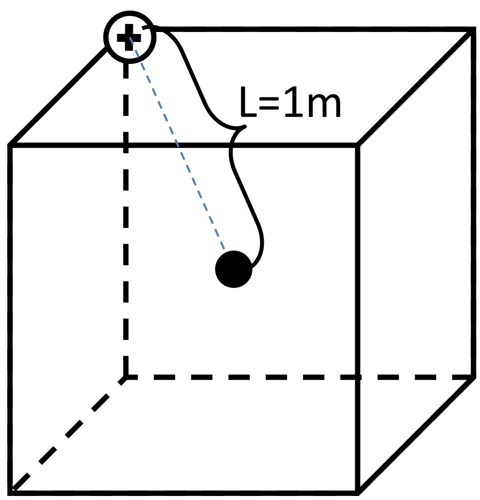

Figure 3.

Then let us calculate the electrostatic potential created by this charge at the center of the cube.

$$\boldsymbol{\varphi_{one\ charge}} = \boldsymbol{k}\frac{\boldsymbol{q}}{\boldsymbol{L}} \approx 8.9 \cdot 10^9 \left(\frac{N \cdot m^2}{C^2}\right) \cdot \frac{2(C)}{1(m)} \approx 1.78 \cdot 10^8 (V)$$

b) Now, we have to take another positive charge, calculate its contribution to the potential etc. But, there is a simpler way. We can use the ***symmetry principle*** which we discussed earlier. As long as all the corners of the cube occupy equivalent positions with respect to the center of the cube, there is no reason to prefer one corner to another. Thus, the contributions of the equal positive charges placed in the corners of the cube to the potential in the cube's center should be equal. So we can just multiply the contribution of one positive charge by 8 – the number of corners.

$$\boldsymbol{\varphi_{total}} = \boldsymbol{\varphi_{one\ charge}} \cdot 8 = 1.78 \cdot 10^{8}(V) \cdot 8$$
$$\approx 1.42 \cdot 10^{7}\ (V)$$

c) Now we can easily calculate the potential energy ***P*** of the charge ***Q=-3C*** placed in the center of the cube:

$$\boldsymbol{P} = \boldsymbol{\varphi_{total}} \cdot \boldsymbol{Q} = 1.42 \cdot 10^{7}\ (V) \cdot (-3)(C) = -4.26 \cdot 10^{7} J$$

Questions:

1. Take a look at the Figure 1. All the attraction forces from the charges in the corners applied to the charge in the cube's center compensate each other. (explain why?). So the charge in the center will be at equilibrium (total net force applied to this charge is zero). Is this equilibrium stable? In other words, what happens if we slightly shift the charge Q from the center toward one of the corners and let it go? Will it return back or continue moving from the center? Explain your answer.

2. How does the potential energy of the charge Q change if we will change the signs of the charges in all the four lower corners of the cube to "minus"?