Homework 5

Electric potential

The potential energy where of two charges separated by a distance *r* is

$$P = k\frac{q_1 \cdot q_2}{r} \quad (1)$$

Let us keep one of the charges, say, q1 fixed and change the charge q2. Since there is a product of the charge magnitudes in the numerator of formula (1), the potential energy will increase or decrease together with the charge magnitude of q2. We can now calculate the potential energy *per unit charge*. For this we will divide the potential energy of the interacting charges q1 and q2 by the magnitude of q2:

$$\frac{P}{q_2} = k\frac{q_1 \cdot q_2}{r} \div q_2 = k\frac{q_1}{r} \quad (2)$$

We can imagine that each point of space around the charge q_1 can be characterized by the potential energy of a positive unit charge in this point. The electrostatic potential energy of a positive unit charge in a certain point is called "***electric potential"*** in this point. The electric potential $\boldsymbol{\varphi}$ created by the point charge $\boldsymbol{q}$ is:

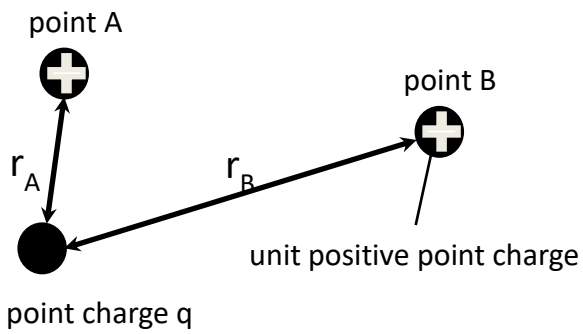
$$\boldsymbol{\varphi = k\frac{q}{r}} \quad (3)$$

If the charge q is negative, the potential will be negative as well.

The formula (3) means that a unit positive point charge placed at the distance r from the charge q will have potential energy $\boldsymbol{\varphi}$. If we will place an arbitrary charge Q at the distance r (instead of a unit charge) then the potential energy of the charge Q can be calculated as:

$$\boldsymbol{P = k\frac{q}{r} \cdot Q = \varphi \cdot Q} \quad \textbf{(4)}$$

As we can see from the formula (3) the potential created by a point charge depends on the distance to the point charge. Difference of potentials taken in points A and B equals to the difference of potential energy of a unit positive charge in these points. As (I hope) you remember difference of potential energy of an object in points B and A is also equal to the work ***we*** have to do to transfer the unit positive charge from point A to point B.



Work (W) we do to move unit positive charge from A to B = φ(at point B)- φ(at pointA)

To find the work for an arbitrary charge Q we have to multiply the potential difference by Q.

$$\boldsymbol{W = Q \cdot (\varphi(B) - \varphi(A)) = Q \cdot \left(\frac{k \cdot q}{r_B} - \frac{k \cdot q}{r_A}\right)} \quad \mathbf{(5)}$$

The potential difference between two points is called ***voltage drop*** between two points or, simply, ***voltage***. W will use a capital U to denote the voltage:

$$\boldsymbol{U_{AB} = \varphi(B) - \varphi(A)} \quad (6)$$

Now, the work needed to move charge Q from point A to point B is :

$$\boldsymbol{W_{AB} = Q \cdot U_{AB}} \quad \mathbf{(7)}$$

The unit of voltage is called Volt (V). Potential difference of 1 V between two points B and A means that we have to perform the work of 1 Joule to move a positive charge of 1 C from point A to point B, so 1V=1 J/C. This unit is named after Italian physicist Alessandro Giuseppe Antonio Anastasio Volta.

A. Volta (1745-1827)

The potential can be calculated not only for a point charge but for any charge system of any geometry**.**

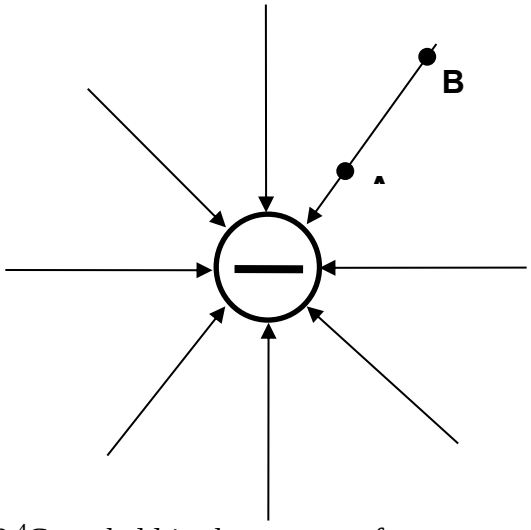
Since the potential is a scalar, to find the total net potential in a certain point of space we have just add the potential created in this point by each of the charges of the system.

It can be measured in any point of space surrounding us. As long as there is a potential difference between two points, positive charge particles placed to the point with higher potential will be "pulled" to the point with lower potential (for the negative particles it is vice-versa). So if we connect two metal balls with different charges the charge will be redistributed among the balls to make the electric potential equal in any point on the surface of the balls.

Problems:

1. We performed work of 6J to move the charge particle with the charge of 0.01C from point A to point B. Find the potential difference between points A and B.

2. There is a point charge of -1C (see picture below). The distance between the charge and the point A is 100m, the distance between the points A and B is also 100m. Find the potential difference between points A and B.



3. 4 charges of same value of 10^{-4}C are held in the corners of a square with the side 1m. Calculate the electrical potential energy of this system.