Homework 4.

Electrostatic potential energy

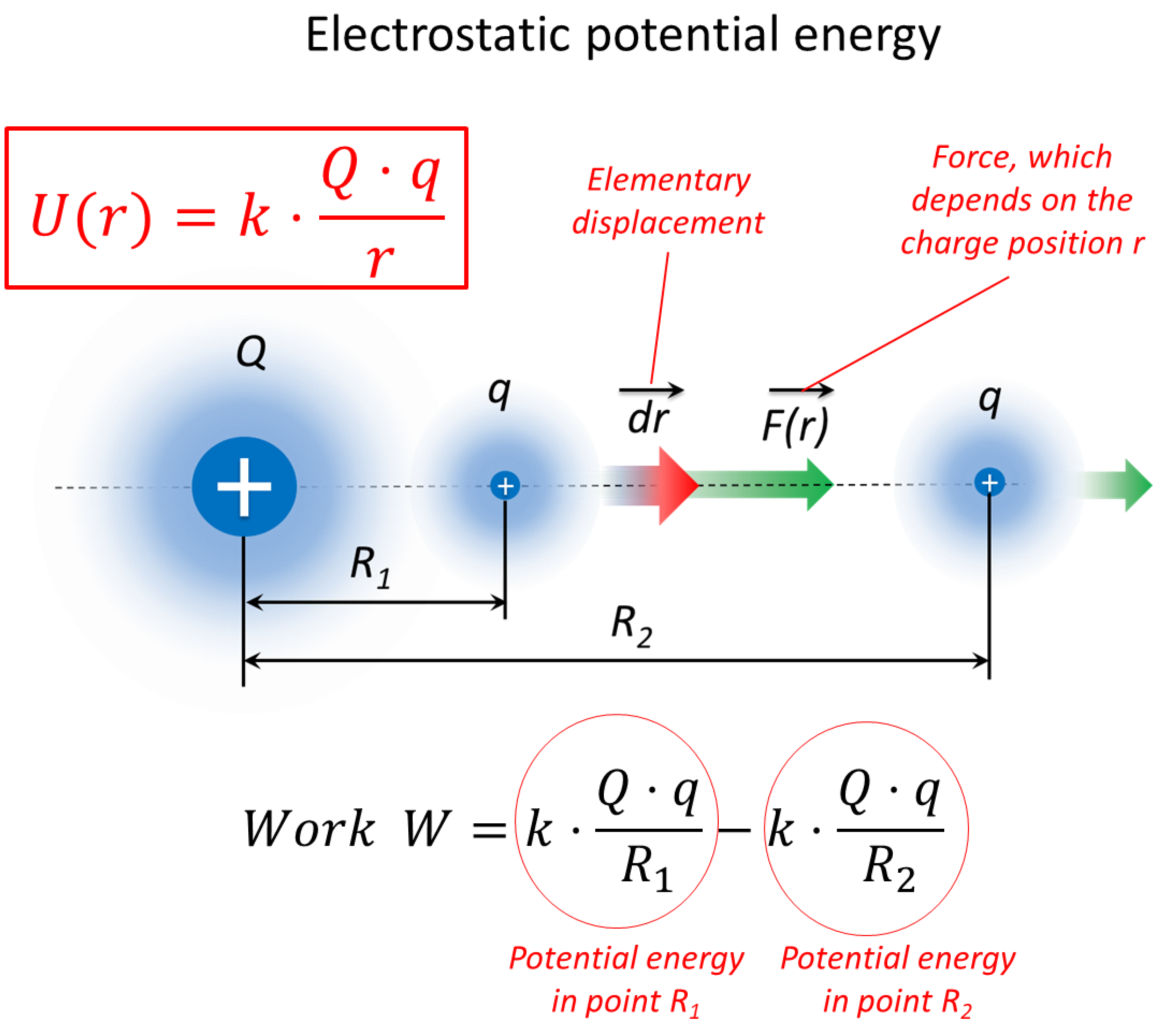
While the electric force between two charges q_1 and $q2$ can be expressed as:

$$F = k\frac{q_1 \cdot q_2}{r^2} \quad (1),$$

where r is the distance between the charges, the potential energy of the charges is

$$U = k\frac{q_1 \cdot q_2}{r} \quad (2).$$

Note that it is just r rather than r^2 in the denominator.



In the picture above a positive charge q (a small object which has a charge q, to be precise) is being pushed away from another positive charge Q. Initially the charge q was at rest, so its kinetic energy is zero. After we let the charge go it will move and the distance between Q and q will increase from R_1 to R_2. In pint R2, the charge q will have both potential and kinetic energy, but the total energy will stay the same as long as the total energy conserves. We can write:

Potential energy in point R_1= Potential energy in point R_2 + Kinetic energy
or:
Potential energy in point R_1- Potential energy in point R_2 = Kinetic energy

But as we, hopefully, remember the change in kinetic energy of an object is equal to the work, done on the object. So that is why

Potential energy in point R_1- Potential energy in point R_2 = Work

k

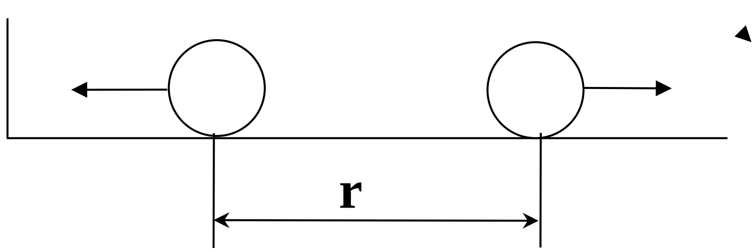
Below is some explanation to the formula for the potential energy:

What do we remember about potential energy?
-Potential energy depends on the position of an object (or objects) – in contrast with the kinetic energy, which depends on the object's velocity.
- The work which is done by the external force on the object as this object is moved from point A to point B is equal to the difference of the object's potential energies in point B and point A.

In physics, the work done by a force is equal to the product of the magnitude of the force and the distance passed by the object along the force direction. But the Coulomb force, in turn, depends on the distance between two charges. Similar situation we had with the elastic force which is proportional to the distance. In case of the elastic force we used average force: it is (force magnitude at the starting point + force magnitude at the end point)/2. Unfortunately, this way does not fork for the Coulomb force, since it is proportional to the square of the inverse distance. But, in the case when R1 and R2 are very close to each other we can take $R_{average}^2 \approx R_1 \cdot R_2$since R_1 is a bit smaler than $R_{average}$ and R_2 is a bit larger. Then we have:

$$Work \approx F_{average} \cdot (R_2 - R_1) = kQq\frac{(R_2-R_1)}{R_{average}^2} = kQq\frac{(R_2-R_1)}{R_1R_2} = k\frac{Qq}{R_1} - k\frac{Qq}{R_1} = U(R_1) - U(R_1).$$

If two charges are of different signs than the potential energy is *negative*. It just means that the closer two charges are, the less is their potential energy. The charges "like" to be as close as possible to reduce their potential energy. So, negative electrostatic potential energy means attraction. For two charges of same sign, the potential energy is *positive* and increases with the distance between them. It means repulsion.
Here is an example:
*Problem: Imagine that we have two identical negatively charged balls (the mass is **M** charge is **q**) are separated by the distance $\mathbf{r}_{before}$. We let the balls go and they start moving. What are the velocities **v** of the balls when the distance between them is $\mathbf{r}_{after}$?*



Solution:
We can try to use Coulomb's law to calculate the force applied to each ball, find acceleration of each ball and, using kinematics formula calculate the time and final velocities. This is a long way, and,

moreover, soon we will meet a serious difficulty – the interaction force and acceleration change with the distance.
There is another, much simpler solution which is based on the energy conservation law. As the balls move away from each other their potential energy decreases, but the kinetic energy of both ball increases. Total energy conserves so the increase of kinetic energy equals to decrease of potential energy. Since the balls are identical, each of them gets halve of the total kinetic energy (symmetry consideration):

$$k\frac{q \cdot q}{r_{after}} + 2M\frac{v^2}{2}$$ - total energy when the distance is 20m

(!) ***we do not multiply the potential energy by 2 since this is "joint" energy of the system of 2 charges. However, we multiply the kinetic energy by 2 since M is the mass of one ball.***

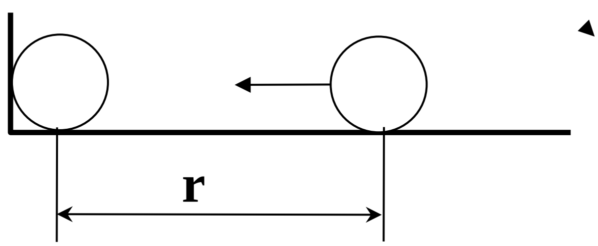
$$k\frac{q \cdot q}{r_{before}}$$ - total energy in the beginning, when the distance is 10m

$$k\frac{q \cdot q}{r_{after}} + 2M\frac{v^2}{2} = k\frac{q \cdot q}{r_{before}}$$ - energy conservation

In this equation we know everything except *v* – so we can easily calculate it.

The homework problems are below:

1. A 2kg positively charged (the charge is 0.001C) small ball is pushed and starts moving toward identical ball with the same charge. The position of the second ball is fixed. When the distance between the balls is 10m, the speed of the first ball is 1m/s. Find the distance at which the moving ball stops?



2. Two identical small metal balls, 1kg each, are charged. The charge of the first is – 0.1C, the charge of the second is +0.3C. The balls are separated by a distance of 5m. After the balls are released they start moving toward each other. Find the velocities of the balls when the distance between them is 2m.

3. Now, the first ball (problem 2) has a mass of 2kg. The other data are the same as in problem 2. Find the velocities of the balls when the distance between them is 2m. (Hint use both energy and momentum conservation laws)