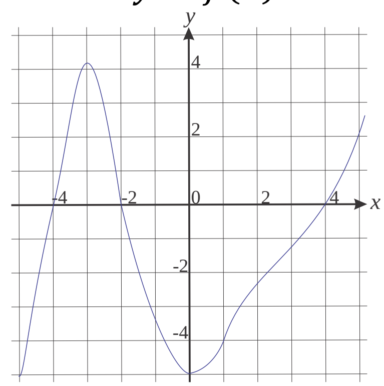
Homework for April 12, 2020.

Algebra.

Review the classwork handout. Solve the following problems.

1. From the picture, find in which interval(s) the function $y = f(x)$
   a. is monotonic
   b. has the same sign



2. Find all possible values of a such that equation $x^2 + ax + 9 = 0$ has two different roots, both of which are less than -1.
3. Draw graphs of the following functions
   a. $y = \left|\frac{1}{x-2} + 1\right|$
   b. $y = \frac{1}{|x|-2} + 1$
4. Solve the following equations
   a. (Skanavi 7.141) $3 \cdot 4^x + \frac{1}{3} \cdot 9^{x+2} = 6 \cdot 4^x - \frac{1}{2} \cdot 9^{x+1}$
   b. (Skanavi 7.143) $\sqrt{\log_x \sqrt{x}} = -\log_x 5$
   c. (Skanavi 7.153) $\frac{\log_2(9-2^x)}{3-x} = 1$
   d. (Skanavi 7.160) $\log_a x + \log_{a^2} x + \log_{a^3} x = 11$
   e. (Skanavi 7.184) $2^{x-1} + 2^{x-4} + 2^{x-2} = 6.5 + 3.25 + 1.625 + \cdots$
   f. (Skanavi 7.190) $9^x + 6^x = 2^{2x+1}$
   g. (Skanavi 7.197) $4^{\log x + 1} - 6^{\log x} - 2 \cdot 3^{\log x^2 + 2} = 0$
   h. (Skanavi 7.299) $(x^2 - x - 1)^{x^2-1} = 1$
   i. (Skanavi 7.304) find integer root: $\log_{\sqrt{x}}(x + 12) = 8\log_{x+12} x$
   j. (Skanavi 7.308) $\log_{x+3}\left(3 - \sqrt{1 - 2x + x^2}\right) = \frac{1}{2}$
5. (Skanavi 7.277) Equation $4^x + 10^x = 25^x$ has a single root. Find this root. Is it positive or negative? Is it larger or less than 1?
6. (Skanavi 7.280) Show that:

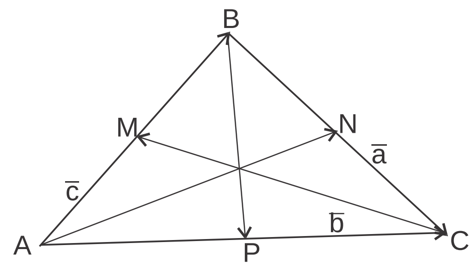
$$\log_3 2 \cdot \log_4 3 \cdot \log_5 4 \cdot \log_6 5 \cdot \log_7 6 \cdot \log_8 7 = \frac{1}{3}$$

Geometry.

Please, complete the previous homework assignments from this year. Review the classwork handout on vectors. Solve the following problems.

Problems.

1. In a triangle ABC, vectors $\overrightarrow{AB}$, $\overrightarrow{AC}$and $\overrightarrow{BC}$ (**c**, **b** and **a**) are the sides. $\overrightarrow{AN}$, $\overrightarrow{CM}$and $\overrightarrow{BP}$are the medians.



   a. Express vectors $\overrightarrow{AN}$, $\overrightarrow{CM}$and $\overrightarrow{BP}$ through vectors **c**, **b** and **a.**

   b. Find the sum of vectors $\overrightarrow{AN}$, $\overrightarrow{CM}$and $\overrightarrow{BP}$**.**

2. Solve the same problem for bisectors $\overrightarrow{AN}$, $\overrightarrow{CM}$and $\overrightarrow{BP}$ in a triangle ABC.

3. Coxeter, Greitzer, problem #9 to Sec. 2.1 (p. 31): How far away is the horizon as seen from the top of a mountain 1 mile high? (Assume the Earth to be a sphere of diameter 7920 miles.)

4. In a rectangle $ABCD$, A_1, B_1, C_1 and D_1 are the mid-points of sides AB, CD, BC and DA, respectively. M is the crossing point of the segments A_1B_1, and C_1D_1, connecting two pairs of midpoints.

   a. Express vector $\overrightarrow{A_1M}$ through $\overrightarrow{AB}$, $\overrightarrow{BC}$ and $\overrightarrow{CD}$.

   b. Prove that M is the mid-point of segments, A_1B_1 and C_1D_1, i.e. $|A_1M| = |MB_1|$ and $|C_1M| = |MD_1|$.

5. In a parallelogram $ABCD$, find $\overrightarrow{AB} + \overrightarrow{BD} - 2\overrightarrow{AD}$.

6. M is a crossing point of the medians in a triangle ABC. Prove that $\overrightarrow{AM} = \frac{1}{3}\left(\overrightarrow{AB} + \overrightarrow{AC}\right)$.

7. For three points, $A(-1,3)$, $B(2,-5)$ and $C(3,4)$, find the (coordinates of) following vectors,

   a. $\overrightarrow{AB} - \overrightarrow{BC}$

   b. $\overrightarrow{AB} + \overrightarrow{CB} + \overrightarrow{AC}$

c. $\overrightarrow{AB} + \frac{1}{2}\overrightarrow{BC} + \frac{1}{3}\overrightarrow{CA}$

8. For two triangles, ABC and $A_1B_1C_1$, $\overrightarrow{AA_1} + \overrightarrow{BB_1} + \overrightarrow{CC_1} = 0$. Prove that medians of these two triangles cross at the same point M.

Trigonometry.

Review the trigonometry classwork handout. Solve the following problems. Some problems are repeated from previous trigonometry homeworks – skip those that you have already solved.

1. Find all x for which,
   a. $\sin x \cos x = \frac{1}{2}$
   b. $\sin x \cos x = \frac{\sqrt{3}}{2}$
2. Find the sum of the following series,

$$S = \cos x + \cos 3x + \cos 5x + \cos 7x + \cdots + \cos 2017x$$

(hint: multiply the sum by $2 \sin x$)

3. Calculate:
   a. $\cos 75° + \cos 15° =$
   b. $\cos\frac{\pi}{12} - \cos\frac{5\pi}{12} =$
4. Prove the following equalities:
   a. $\frac{1}{\sin\alpha} + \frac{1}{\tan\alpha} = \cot\frac{\alpha}{2}$
   b. $\sin^2\left(\frac{7\pi}{8} - 2\alpha\right) - \sin^2\left(\frac{9\pi}{8} - 2\alpha\right) = \frac{\sin 4\alpha}{\sqrt{2}}$
   c. $(\cos\alpha - \cos\beta)^2 + (\sin\alpha - \sin\beta)^2 = 4\sin^2\frac{\alpha-\beta}{2}$
   d. $\frac{\cot^2 2\alpha - 1}{2\cot 2\alpha} - \cos 8\alpha \cot 4\alpha = \sin 8\alpha$
   e. $\sin^6\alpha + \cos^6\alpha + 3\sin^2\alpha\cos^2\alpha = 1$
   f. $\frac{\sin 6\alpha + \sin 7\alpha + \sin 8\alpha + \sin 9\alpha}{\cos 6\alpha + \cos 7\alpha + \cos 8\alpha + \cos 9\alpha} = \tan\frac{15\alpha}{2}$
   g. $\sin^6\alpha + \cos^6\alpha = \frac{5 + 3\cos 4\alpha}{8}$
   h. $16\sin^5\alpha - 20\sin^3\alpha + 5\sin\alpha = \sin 5\alpha$

i. $\frac{\cos 64° \cos 4° - \cos 86° \cos 26°}{\cos 71° \cos 41° - \cos 49° \cos 19°}$

j. $\sin 20° \sin 40° \sin 60° \sin 80° = \frac{3}{16}$

k. $\frac{1}{\sin 10°} - \frac{\sqrt{3}}{\cos 10°} = 4$

5. Simplify the following expressions:

l. $\sin^2\left(\frac{\alpha}{2} + 2\beta\right) - \sin^2\left(\frac{\alpha}{2} - 2\beta\right)$

m. $2\cos^2 3\alpha + \sqrt{3}\sin 6\alpha - 1$

n. $\cos^4 2\alpha - 6\cos^2 2\alpha \sin^2 2\alpha + \sin^4 2\alpha$

o. $\sin^2(135° - 2\alpha) - \sin^2(210° - 2\alpha) - \sin^2 195° \cos(165° - 4\alpha)$

p. $\frac{\cos 2\alpha - \cos 6\alpha + \cos 10\alpha - \cos 14\alpha}{\sin 2\alpha + \sin 6\alpha + \sin 10\alpha + \sin 14\alpha}$