Homework for November 17, 2019.

Algebra.

Review the previous classwork handout. Solve the remaining problems from the previous homework assignments and classwork exercises. Try solving the following problems.

1. Using the method of mathematical induction, prove the following equality,

$$\sum_{k=0}^{n} k \cdot k! = (n+1)! - 1$$

2. Put the sign $<$, $>$, or $=$, in place of … below,

$$\frac{n+1}{2} \ldots \sqrt[n]{n!}$$

3. Consider the quadratic equation $x^2 = 7x + 1$. Find a continued fraction corresponding to a root of this equation.

4. Find the value of the continued fraction given by

$$\{1,2,3,3,3, \ldots\} = 1 + \cfrac{1}{2 + \cfrac{1}{3 + \cfrac{1}{3 + \cfrac{1}{3 + \cdots}}}}$$

5. Find the set of all values of x for which the following expression makes sense: $\sqrt{25 - x^2} + \frac{4}{x-2} - \frac{1}{x}$.

6. Is the set of all squares $S = \{x : x = a^2, a \in N\}$ countable?

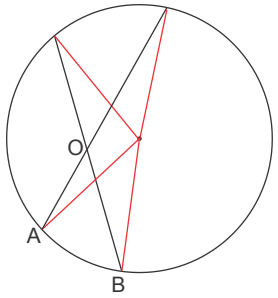
7. It is known that the set of all real numbers is not countable. Let us consider the following set $S = \{x : 0 \leq x \leq 1\}$. Is this set countable? Can you prove it?

Geometry.

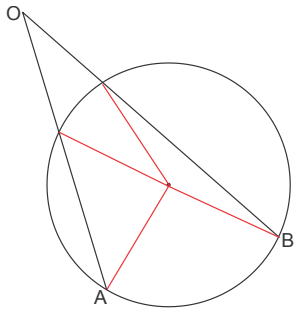
Review the last classwork handout on inscribed angles and quadrilaterals. Go over the proof of Ptolemy's theorem. Solve the unsolved problems from previous homework. Try solving the following problems.

Problems.

1. Prove that an angle whose vertex lies inside a disk is measured by a semi-sum of the two arcs, one of which is intercepted by this angle, and the other by the angle vertical to it.



2. Prove that an angle whose vertex lies outside a disk and whose sides intersect the circle, is measured by a semi-difference the two intercepted arcs.



3. A point P is chosen on the chord AB of a circle with the center O and radius R. Show that $|AP|\times|PB| = R^2 - |OP|^2$.

4. Consider all triangles with a given base and given altitude corresponding to this base. Prove that among all these triangles the isosceles triangle has the biggest angle opposite to the base.