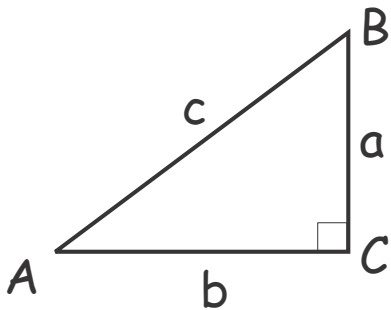


September 15, 2019

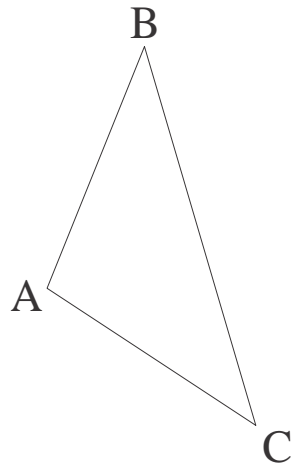
## Geometry.

### Baseline revue test. Geometry.

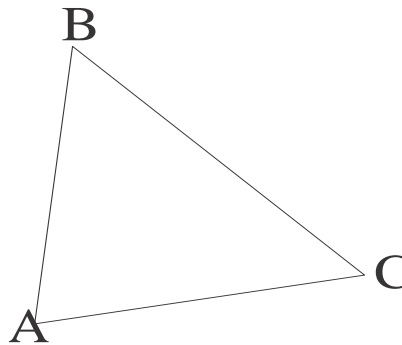
1. List undefined terms (primitives) of geometry.
2. Give definition of (i) a segment (ii) a circle.
3. List three congruence tests for triangles.
4. State and prove the Pythagorean theorem.



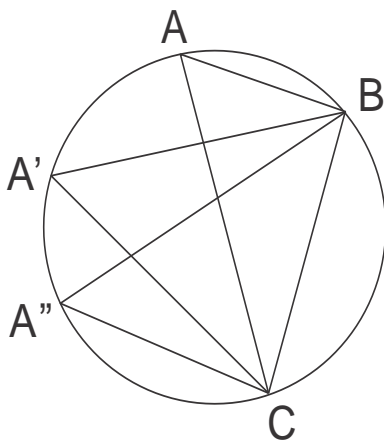
5. List formulas for the area of a triangle that you know (sides are  $a$ ,  $b$ ,  $c$ , heights to these sides are  $h_a$ ,  $h_b$ ,  $h_c$  respectively, the radius of the inscribed circle is  $r$ , that of the circumscribed circle is  $R$ ).
6. Using a compass and a ruler, draw a circle circumscribed around a given triangle.



7. Using a compass and a ruler, draw a circle inscribed into a given triangle.

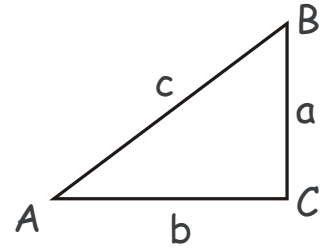


7. Which of the angles  $BAC$ ,  $BA'C$ ,  $BA''C$  is the largest? Which is the smallest?

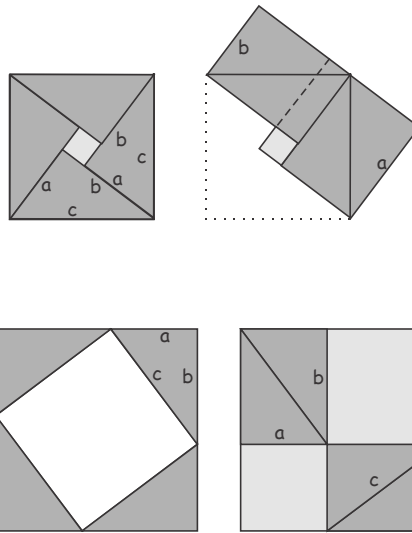


## Recap: The Pythagorean Theorem.

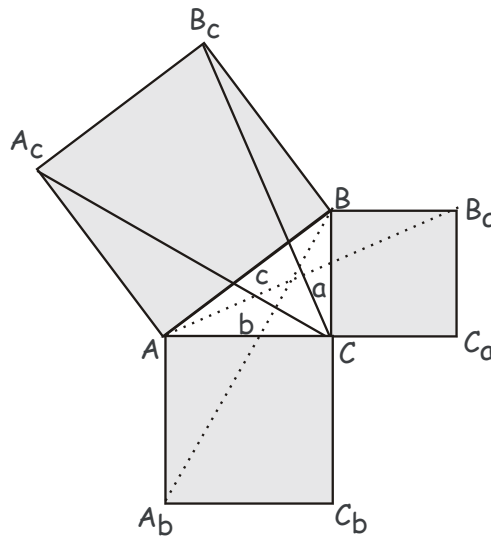
**Theorem.** In a right triangle with legs  $a$  and  $b$  and hypotenuse  $c$ ,  $a^2 + b^2 = c^2$ .



**Proof 1.** Perhaps, the most elegant are the algebra-free proofs by dissection, as shown in Figures below.



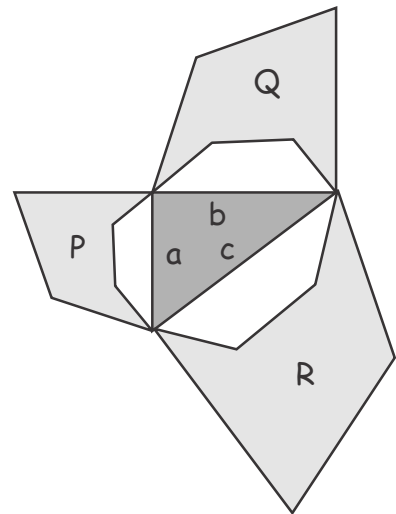
**Proof 2.** Perhaps, the most famous proof is that by Euclid, although it is neither the simplest, nor the most elegant. It is illustrated in Fig. 3 below.



### Generalized Pythagorean Theorem.

If three similar polygons, P, Q and R with areas  $S_P$ ,  $S_Q$  and  $S_R$  are constructed on legs  $a$ ,  $b$  and hypotenuse  $c$ , respectively, of a right triangle, then,

$$S_P + S_Q = S_R$$



### Recap: Inequalities in triangles.

**Definition.** The angle supplementary to an angle of a triangle is called an exterior angle of this triangle.

**Theorem 1.** An exterior angle of a triangle is greater than each of the interior angles not supplementary to it.

**Theorem 2a.** In any triangle,

- the angles opposite to congruent sides are congruent
- the sides opposite to congruent angles are congruent

**Theorem 2b.** In any triangle,

- The angle opposite to a greater side is greater
- The side opposite to a greater angle is greater

**Theorem 3 (triangle inequality).** In any triangle, each side is smaller than the sum of the other two sides, and greater than their difference,

$$|AB| - |BC| < |AC| < |AB| + |BC|$$

**Theorem 4 (corollary).** The line segment connecting any two points is smaller than any broken line connecting these points.

### Recap: Parallelogram. Central Symmetry.

**Definition.** A quadrilateral whose opposite sides are pairwise parallel is called a parallelogram.

**Theorem 1a.** In a parallelogram, opposite sides are congruent.

**Theorem 1b.** In a quadrilateral, if the opposite sides are congruent, then this quadrilateral is a parallelogram.

**Theorem 1c.** In a quadrilateral, if two opposite sides are parallel and congruent, then this quadrilateral is a parallelogram.

**Theorem 2a.** In a parallelogram, opposite angles are congruent.

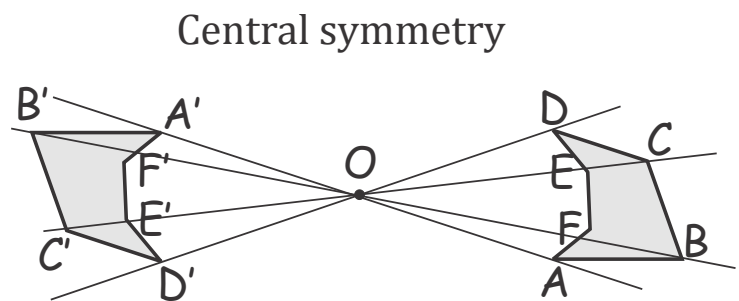
**Theorem 2b.** In a quadrilateral, if opposite angles are congruent, then this quadrilateral is a parallelogram.

**Theorem 3a.** In a parallelogram, diagonals bisect each other.

**Theorem 3b.** In a quadrilateral, if the diagonals bisect each other, then this quadrilateral is a parallelogram.

### Recap: Central Symmetry.

**Definition.** Two points  $A$  and  $A'$  are symmetric with respect to a point  $O$ , if  $O$  is the midpoint of the segment  $AA'$ .



**Definition.** Two figures are symmetric with respect to a point  $O$ , if for each point of one figure there is a symmetric point belonging to the other figure, and vice versa. The point  $O$  is called the center of symmetry.

Symmetric figures are congruent and can be made to coincide by a 180 degree rotation of one of the figures around the center of symmetry.

Diagonals of a parallelogram divide it into two pairs of symmetric triangles with respect to the intersection point of its diagonals. The parallelogram is symmetric to itself about this point.