MATH 8
ASSIGNMENT 1: COMBINATORICS REVIEW

SEPTEMBER 15, 2019

Welcome to the new semester at SchoolNova!!

This Fall, we plan to study the following topics:

- Review of combinatorics. Binomial formula.
- Number theory: Euclid's algorithm, congruences and modular arithmetic.

We will try to do much of the homework in class so that you do not need to spend too much time on it at home. As usual, all HW assignments and other information will be posted online at `http://www.schoolnova.com` (click on Homeworks in the navigation bar on top).

I ask that each student bring a notebook (preferably quad ruled), pencils and a folder or binder to keep old assignments — you will need them!

We also plan to participate in two math competitions: Math Kangaroo and American Math Contests (AMC). More details will be given later.

If you have any questions, please contact me by email: `Helmut.Strey@stonybrook.edu`.

Main formulas of combinatorics

- The number of ways to order k items is
$$k! = k(k-1)\cdots 2\cdot 1$$
- The number of ways to choose k items out of n if the order matters is
$${}_nP_k = n(n-1)\dots(n-k+1) = \frac{n!}{(n-k)!}$$
- The number of ways to choose k items out of n if the order does not matter is
$${}_nC_k = \frac{n(n-1)\cdots(n-k+1)}{k(k-1)\cdots 1} = \frac{n!}{(n-k)!k!}$$

These numbers are the ones that appear in Pascal triangle and in many other problems:

${}_nC_k =$ The number of paths on the chessboard going k units up and $n-k$ to the right

$=$ The number of words that can be written using k zeros and $n-k$ ones

Problems

1. There are 25 prisoners in Cannibal's dungeons.
   (a) In how many ways can he choose three of them for breakfast, lunch and dinner?
   (b) In how many ways can he choose three of them to let them go free?

2. A party of 5 men sits at a bar, which offers 17 different cocktails.
   (a) If each man wants to order a cocktail, how many possible combinations are there?
   (b) If each man wants to order a cocktail so that all of them have different cocktails, how many possible combinations are there?
   (c) How many way there are to choose 5 cocktails out of 17 (all 5 different, but the order doesn't matter)?

3. Remember that a poker hand is a selection of 5 cards out of a 52-card deck (4 suits, 13 card ranks in each suit).

   Count the number of possible poker hands with the following conditions:
   (a) All possible hands.
   (b) Hands that only contain spades
   (c) Containing 4 cards of one suit and one card of another.

4. We toss a coin 100 times.
   (a) What is the probability of obtaining all tails? exactly 2 heads? at least 1 head?
   (b) Same question for an unfair coin, which gives heads with probability $p = 0.45$ and tails with probability $q = 0.55$.

5. In roulette, there are 37 slots: 18 red, 18 black, and zero (which has no color). Find the probability of getting a sequence of 10 reds in a row; of having exactly 5 reds in a sequence of 10 rolls.

6. A certain machine produces identical parts. The probability that it will produce a defective part is $p = 0.1\%$.
   (a) Find the probability that out of n parts, there will be no defective ones.
   (b) What is the probability that out of n parts, there will be exactly 1 defective part? exactly 2? not more than 2?

7. A staircase consists of 7 steps not counting upper and lower landings. Going downstairs one can jump over some of steps (even all seven). In how many ways one can descend this staircase?

8. How many ways there are to permutate the letters of the word SchoolNova?

9. A *monomial* is a product of powers of variables, i.e. an expression like x^3y^7.
   (a) How many monomials in variables x, y of total degree of exactly 15 are there? (Note: this includes monomials which only use one of the letters, e.g. x^{15}.)
   (b) Same question about monomials in variables x, y, z.[Hint: if you write 15 letters in a row, you need to indicate where x's end and y's begin — you can insert some kind of marker to indicate where it happens.]
   (c) How many monomials in variables x, y of degree at most 15 are there?
   *(d) How many monomials in variables x, y, z of degree at most 15 are there?