

MATH 8: FINAL TEST

1. Compute $(2x + y)^7 - (2x - y)^7$. Use binomial theorem.
2. A full house is a collection of five cards in poker that consists of a three-of-a-kind and a two-of-a-kind. Calculate the number of possible full houses that one can make from a standard 52 card deck.
3. Prove: $\neg(p \Rightarrow q) \Leftrightarrow (p \wedge \neg q)$.
4. Write the following statements using logic connectives and quantifiers:
 - (a) All linguists know mathematics
 - (b) Some linguists don't know mathematics
 - (c) No one but a linguist likes mathematicsPlease use the following notation:
 - P – set of all people
 - $L(x)$ — x is a linguist
 - $M(x)$ — x loves mathematics
5. Prove that $ABCD$ is a parallelogram if and only if its diagonals bisect each other. [Do not forget – you need to provide two proofs!]
6. Given triangle $\triangle ABC$, complete a straightedge-compass construction of a circle that passes through A, B, C (circumscribed circle).
7. Given triangle $\triangle ABC$, complete a straightedge-compass construction of a circle that touches sides of the triangle (inscribed circle).
8. Find an inverse of 9 (mod 13). Use Euclid's algorithm.
9. Find the solution of the following equation in whole numbers: $11x + 19y = 2$.
10. Solve the following system of congruences:

$$x \equiv 3 \pmod{13}$$

$$x \equiv 5 \pmod{9}$$