MATH 8: ASSIGNMENT 1

JANUARY 12, 2020

1. Final Review

Here is a collection of problems covering past material from the academic year. **Use these as a guide to review and study the past homework sheets** - in particular, don't just solve the problems, but review each of the concepts you use either in your notes or in the past homework sheets themselves (or both).

2. Homework

1. A full house is a collection of five cards that consists of a three-of-a-kind and a two-of-a-kind. Calculate the number of possible full houses that one can make from a standard 52 card deck.

2. Consider a function of the form $f(x) = mx + b$. Is it possible to find such a function such that $f(0) = f(1)$ and $f(2) = 0$? Is it possible to find such a function such that $f(0) = f(1) = 0$ and $f(2) = 1$?

3. Simplify $\neg(A \implies B)$

4. Simplify $(A \vee B) \implies (A \wedge B)$

5. Your flight to Melbourne is scheduled to stop at an airport in Sydney at noon; you must then transfer to a plane scheduled to depart for Melbourne at 1pm. You know that your flight to Sydney is going to be randomly delayed anywhere from 1 hour to 3 hours; you also know that the flight from Sydney to Melbourne is randomly delayed by anywhere from 1 hour to 2 hours. What is the probability that you will be able to make your connection?

6. Prove that if $x^3 \equiv x \mod 11$, then $x \equiv 1 \mod 11$.

7. Prove that if $x^3 \equiv x \mod 101$, then $x \equiv 1 \mod 101$.

8. Let C be a circle of radius 1 centered at point O. Let A, B be points on the circle C. Let D be a circle that goes through O and is tangent to C; additionally, let D be such that it intersects the lines AO and BO at points X, Y, where X, Y are different from O. Calculate the distance XY.

9. You select two distinct numbers at random from 0 to 100, and you call them x, y respectively. What is the probability that you can find a positive integer k with $0 < k < 10$ such that $y - x \equiv k \mod 100$?

10. Let two strings of letters be called *incompatible* if they do not share a common prefix - i.e., if they do not have letters in common at the beginning. For example, $xmtr$ and $rsab$ are incompatible, but $xmtr$ and $xsab$ are compatible (with shared prefix x). Describe, with proof, the largest possible set of incompatible letter strings.