# MATH 8: EUCLIDEAN GEOMETRY 2

JANUARY 19, 2020

## 1. Locus & Circle

Consider some logical property of points - i.e., some statement that can be either true or false for any given point on the plane. Such examples include: the point's distance from some given point $X$ is 1; the point is inside some given triangle $\triangle ABC$; the point is between two given parallel lines $l$, $m$. The set of all points for which a property holds true is called the locus of points satisfying the property.

As your first example, the locus of points that are a fixed distance from a given point $O$ is called a circle. The point $O$ is called the center of the circle, and the fixed distance is called the radius distance. A line segment from $O$ to some point on the circle is called a radius of the circle. Note that all radii of a circle are congruent.

Two different circles are congruent if they have the same radius distance; equivalently, if their respective radii are congruent.

Circles are vastly useful objects, and it will be enormously helpful to be comfortable with the concept of what a circle is. Fortunately, circles are familiar to most of us. They do have less obvious properties, but we will prove those later. For now, the basic definition will suffice, as well as the following useful definition related to circles: given a circle with center $O$, a line segment from $O$ to a point on the circle is called a radius of the circle. Note that all radii of a circle are congruent. Isn't that exciting!

## 2. Right Angle & Right Triangle

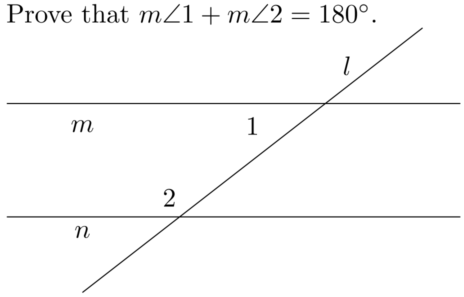
An angle whose measure is $90°$ is called a right angle; a triangle where one of the angles is a right angle is called a right triangle. The side opposite the right angle is called the hypotenuse of the triangle; the other two sides are called the legs.

## 3. Homework

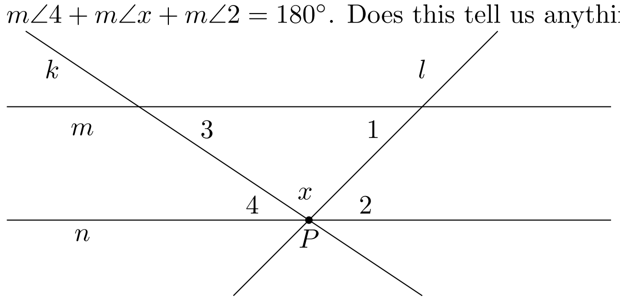
**Note that you may use all results that are presented in previous sections, assignments, and homework problems.** This means that you may use Theorem 3, for example, if you find it a useful logical step in your proof.

**1.** (Triangle) The goal of this problem is to prove Theorem 8; we will approach it via some helpful first steps, which are also in themselves good exercises in logic and geometry.

(a) Consider lines $l$, $m$, and $n$ are such that $m \parallel n$ and $l$ intersects them both, as shown below. Prove that $m\angle 1 + m\angle 2 = 180°$.




(b) Consider lines $k$, $l$, $m$, and $n$ such that $m \parallel n$ and $k$, $l$, and $n$ all intersect at $P$. Notice that $m\angle 4 + m\angle x + m\angle 2 = 180°$. Does this tell us anything about the sum $m\angle 1 + m\angle x + m\angle 3$?




(c) Prove theorem 8.

**2.** (Locus) Let $A$, $B$ be some two points on the plane. A point is said to be equidistant from $A$, $B$ if its distance to $A$ is the same as its distance to $B$; in other words, a point $X$ is equidistant from $A$, $B$ if $\overline{AX} \cong \overline{BX}$. Let $L$ be the locus of points equidistant from $A$, $B$.

(a) Let $M$ be the point in $L$ that is on the line segment $\overline{AB}$. Can you describe the location of $M$? How does the distance $AB$ compare to the distance $AM$? What about the distance $BM$?

(b) Let $P$ be a point in $L$ other than $M$. Determine the measure of the angle $\angle AMP$.

(c) Let $Q$ be a point in $L$ other than $M$ and $P$. Prove that $m\angle PMQ$ is either $0°$ or $180°$.

(d) Deduce that $L$ is a line.

(e) Let $\omega$, $\lambda$ be congruent circles whose centers are $A$, $B$ respectively, and whose radius distance is larger than the distance $AB$. Suppose these two circles intersect at two distinct points $X$, $Y$. Prove that $\overleftrightarrow{XY} = L$.

**3.** (Equilateral) Suppose $\triangle ABC$ is a triangle.

(a) Prove that all three sides of $\triangle ABC$ are congruent if and only if all three angles measure $60°$. Such a triangle is called equilateral.

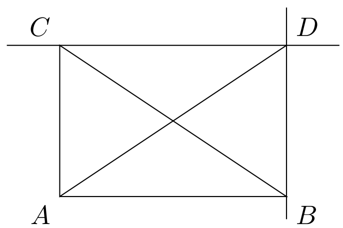
(b) Let $X$ be on $\overline{AB}$, $Y$ on $\overline{BC}$, $Z$ on $\overline{AC}$ such that $\overline{AX} \cong \overline{BY} \cong \overline{CZ}$. Prove that if $\triangle ABC$ is equilateral, then $\triangle XYZ$ is equilateral. Make a guess as to whether the converse is true.

**4.** (Triangle Rights) Let $\triangle ABC$ be a right triangle with right angle $\angle A$, and let $D$ be the intersection of the line parallel to $\overline{AB}$ through C with the line parallel to $\overline{AC}$ through B.

(a) Prove $\triangle ABC \cong \triangle DCB$

(b) Prove $\triangle ABC \cong \triangle BDA$

(c) Prove that $\overline{AD}$ intersects $\overline{BC}$ at the midpoint of $\overline{BC}$.




**5.** (Pentagon!) A *pentagon* is a figure consisting of five line segments and five angles; a *regular pentagon* is one where all the sides are congruent, and all the angles are congruent (see below).

(a) Find, with proof, the sum of the angle measures of a regular pentagon.

(b) Deduce the angle measure of a single angle of a regular pentagon.

(c) Prove that $\overline{AC} \parallel \overline{DE}$.