MATH 8
FALL REVIEW
DECEMBER 15, 2019

1. How many ways are there to choose a committe of three people from a group of six people? What if one of the committee members must be selected to be president?
2. If I never ever do laundry, and I always instead fold my day's outfit and put it back into my wardrobe at the end of the day, if I have ten outfits in my wardrobe, in how many ways can I choose outfits for the week? (I must wear exactly one outfit every day).
3. How many subsets are there of an n-element set?
4. How many ways are there to place two rooks on a cheesboard so that they are not attacking each other? Assume the rooks are the same color. [For those of you who are unfamiliar, a chessboard is an 8x8 grid of squares, and rooks are pieces that can occupy any one of these individual squares, and may attack any other piece that's in the same row or column of the board as itself.]
5. How many five letter "words" are there such that the letters are in alphabetical order? (Here a "word" is any sequence of letters from the alphabet a through z.)
6. Andrew has 7 pieces of candy, and Tim has 9 (all different). They want to trade 5 pieces of candy. How many wasy are there for them to do it?
7. A *monomial* is a product of powers of variables, i.e. an expression like x^3y^7.
 (a) How many monomials in variables x, y of total degree of exactly 15 are there? (Note: this includes monomials which only use one of the letters, e.g. x^{15}.)
 (b) Same question about monomials in variables x, y, z.[Hint: if you write 15 letters in a row, you need to indicate where x's end and y's begin — you can insert some kind of marker to indicate where it happens.]
 (c) How many monomials in variables x, y of degree at most 15 are there?
 *(d) How many monomials in variables x, y, z of degree at most 15 are there?
8. Let p be prime.
 (a) Show that each of the binomial coefficients ${}_pC_k$, $1 \leq k \leq p-1$, is divisible by p.
 (b) Show that if a, b are integer, then $(a+b)^p - a^p - b^p$ is divisible by p.
9. Which coefficient in the n^{th} row of Pascal's Triangle is the largest? Can you prove why?
10. Prove that $\binom{n}{1} + 2\binom{n}{2} + 3\binom{n}{3} + 4\binom{n}{4} + ... + n\binom{n}{n} = n2^{n-1}$
11. Prove that $(A \implies B) \wedge (B \implies A)$ is equivalent to $A \iff B$
12. A teacher tell the student "If you do not take the final exam, you get an F". Does it mean that
 (a) If the student does take the final exam, he will not get an F
 (b) If the student does not get an F, it means he must have taken the final exam.
13. Use the truth tables to prove *De Morgan's laws*

$$\neg(A \wedge B) \iff (\neg A) \vee (\neg B)$$
$$\neg(A \vee B) \iff (\neg A) \wedge (\neg B)$$

14. The following statement is sometimes written on highway trucks:
 If you can't see my windows, I can't see you.
 Can you write an equivalent statement without using word "not" (or its variations such as "can't").
15. Prove the following logical equivalence: $\neg(p \implies q) \iff (p \wedge \neg q)$
16. Given logical statements m, p, q, let a denote the combined statement $(m \wedge p) \vee (\neg m \wedge q)$. In other words, $a \iff ((m \wedge p) \vee (\neg m \wedge q))$. Prove the following:
 (a) If m is true, then $a \iff p$
 (b) If m is false, then $a \iff q$
17. Write each of the following statements using only quantifiers, arithmetic operations, equalities and inequalities. In all problems, letters x, y, z stand for a variables that takes real values, and letters $m, n, k, \ldots$ stand for variables that take integer values.
 (a) Equation $x^2 + x - 1$ has a solution

(b) Inequality $y^3 + 3y + 1 < 0$ has a solution
(c) Inequality $y^3 + 3y + 1 < 0$ has a positive real solution
(d) Number 100 is even.
(e) Number 100 is odd
(f) For any integer number, if is is even, then its square is also even.