MATH 8
ASSIGNMENT 1: COMBINATORICS REVIEW

SEPTEMBER 15, 2019

Welcome to Fall 2019 at SchoolNova!!

This Fall, we plan to study the following topics:

- Review of combinatorics. Binomial formula.
- Logic and proofs

We will try to do much of the homework in class so that you do not need to spend too much time on it at home. As usual, all HW assignments and other information will be posted online at
`http://www.schoolnova.org/nova/homeworks`

We also plan to participate in two math competitions: Math Kangaroo and American Math Contests (AMC). Participation is optional, but if you are interested or have any questions about the competitions, let me know!

The Math Kangaroo (`http://www.mathkangaroo.org`) is an international math competition with a range of levels available for all ages/grades K-12. The contest is in March; details of the registration will be given later in the year.

AMC (`http://amc.maa.org/`) is the "official" American Math Olympiad: it is the first level of the competition that eventually leads to the selection of US team for International Math Olympiad. AMC 8 is intended for students in grades 8 and below; AMC 10 for students in grades 10 and below. The AMC 8 takes place in November; the AMC 10 in late January/early February of next year. You do not have to register individually – simply let me know if you are interested.

Combinatorics!

1. There are four sheep in a pen, each with a distinct color of wool. I want to make a sweater from their wool (I will ask the sheep nicely before shearing them).
   (a) If I want a sweater with two colors of wool, how many possible pairs of colors are there for me to select from?
   (b) If I want a sweater primarily of one color with a trim made of a second color, how many possible ways are there for me to pick these two colors?

2. 12 sentient frogs wish to select 1 leader and then a board of administration. The board is to be comprised of 3 frogs, and the leader may not be on the board. How many possible ways are there to fill the positions? (Assume all the frogs are distinct, with distinct personalities).

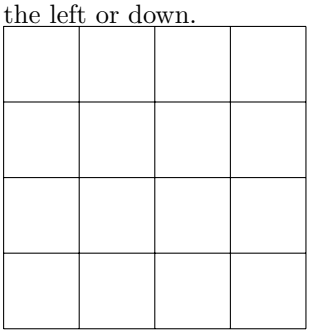
3. In a meeting of 25 people, every one of them shakes hands once with every other. How many handshakes took place altogether?

4. How many ways are there to select two black cards and three red cards from a (standard) deck of cards? (In a standard deck there are 52 cards, with exactly half red and the other half black).

5. How many two-digit numbers are there where the first digit is strictly larger than the second digit? Examples: 42, 61, and 10 are such integers, but 18, 44, and 56 are not.
What about three-digit numbers? In this case, count how many there are such that each digit is strictly smaller than the digit to the left.

6. Suppose there are ten windows in a line, each of them open. A bee wants to fly into the house to grab a snack, then fly back out before any human notices. However, there is a bit of wind - the windows happen to be lined up on a wall that runs west to east, and due to the wind the bee will have to fly out a window that's to the east of the one it came in from. How many windows down it flies, however, is up to the bee. In how many ways can the bee select and entrance and exit window?

7. How many different paths are there on 4×4 chessboard connecting the lower left corner with the upper right corner? What about 5×5? The path should always be going to the right or up, never to the left or down.



8. How many "words" of length 8 one can write using only letters U and R, including exactly 4 U's and 4 R's? (a "word", for our purposes, is any sequence of letters.)

***9.** You have 10 books which you want to put on 2 bookshelves. How many ways are there to do it (order on each bookshelf matters)?

Miscellaneous!

1. Let a, b, c be distinct positive integers. Is it possible that $\frac{1}{a} + \frac{1}{b} + \frac{1}{c} = 2$? How about $\frac{1}{a} + \frac{1}{b} + \frac{1}{c} = 1$?

2. Let n be a positive integer greater than 10. Is it possible for n to have more than $\frac{n}{2}$ factors?

3. (a) Determine the distance between the two intersection points of the graphs of $y = x^2$ and $y = 2$.
   (b) Determine the distance between the two intersection points of the graphs of $y = x^2$ and $y = x+2$.
   (c) Write the square root of 32 in simplified form as $a\sqrt{b}$ for positive integers a and b (*simplified* means a should be as large as possible).