Math 7

Solving Quadratic Equations. Quadratic Formula.

We can use completing the square to derive a general formula to solve quadratic equations

$$ax^2 + bx + c = 0$$

Here is how it works

If you complete the square for the general equation $ax^2 + bx + c = 0$, you can derive the quadratic formula.

Step 1 Write $ax^2 + bx + c = 0$ so the coefficient of x^2 is 1.

$$ax^2 + bx + c = 0$$

$$x^2 + \frac{b}{a}x + \frac{c}{a} = 0 \quad \text{Divide each side by } a.$$

Step 2 Complete the square.

$$x^2 + \frac{b}{a}x = -\frac{c}{a} \quad \text{Subtract } \frac{c}{a} \text{ from each side.}$$

$$x^2 + \frac{b}{a}x + \left(\frac{b}{2a}\right)^2 = -\frac{c}{a} + \left(\frac{b}{2a}\right)^2 \quad \text{Add } \left(\frac{b}{2a}\right)^2 \text{ to each side.}$$

$$\left(x + \frac{b}{2a}\right)^2 = -\frac{c}{a} + \frac{b^2}{4a^2} \quad \text{Write the left side as a square.}$$

$$\left(x + \frac{b}{2a}\right)^2 = -\frac{4ac}{4a^2} + \frac{b^2}{4a^2} \quad \text{Multiply } -\frac{c}{a} \text{ by } \frac{4a}{4a} \text{ to get like denominators.}$$

$$\left(x + \frac{b}{2a}\right)^2 = \frac{b^2 - 4ac}{4a^2} \quad \text{Simplify the right side.}$$

Step 3 Solve the equation for x.

$$\sqrt{\left(x + \frac{b}{2a}\right)^2} = \pm\sqrt{\frac{b^2 - 4ac}{4a^2}} \quad \text{Take square roots of each side.}$$

This step uses the property $\sqrt{\frac{m}{n}} = \frac{\sqrt{m}}{\sqrt{n}}$.

$$x + \frac{b}{2a} = \pm\frac{\sqrt{b^2 - 4ac}}{2a} \quad \text{Simplify the right side.}$$

$$x = -\frac{b}{2a} \pm \frac{\sqrt{b^2 - 4ac}}{2a} \quad \text{Subtract } \frac{b}{2a} \text{ from each side.}$$

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a} \quad \text{Simplify.}$$

Quadratic Formula

If $ax^2 + bx + c = 0$, and $a \neq 0$, then

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

Example 1. Suppose $2x^2 + 3x - 5 = 0$. Then $a = 2, b = 3$, and $c = -5$. Therefore

$$x = \frac{-(3) \pm \sqrt{(3)^2 - 4(2)(-5)}}{2(2)}$$

Example 2.

What are the solutions of $x^2 - 8 = 2x$? Use the quadratic formula.

$x^2 - 2x - 8 = 0$ — Write the equation in standard form.

$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$ — Use the quadratic formula.

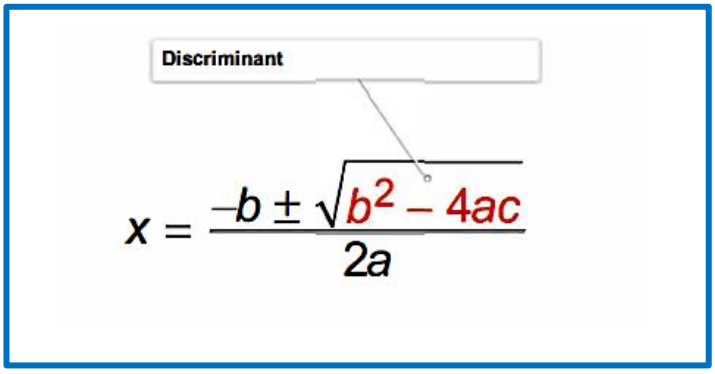
$x = \frac{-(-2) \pm \sqrt{(-2)^2 - 4(1)(-8)}}{2(1)}$ — Substitute 1 for a, -2 for b, and -8 for c.

$x = \frac{2 \pm \sqrt{36}}{2}$ — Simplify.

$x = \frac{2+6}{2}$ or $x = \frac{2-6}{2}$ — Write as two equations.

$x = 4$ or $x = -2$ — Simplify.

Quadratic equations can have two, one, or no real-number solutions. Before you solve a quadratic equation, you can determine how many real-number solutions it has by using the discriminant. The **discriminant** is the expression under the radical sign in the quadratic formula.



Using the discriminant

Discriminant	$b^2 - 4ac > 0$	$b^2 - 4ac = 0$	$b^2 - 4ac < 0$
Example	$x^2 - 6x + 7 = 0$ The discriminant is $(-6)^2 - 4(1)(7) = 8$, which is positive.	$x^2 - 6x + 9 = 0$ The discriminant is $(-6)^2 - 4(1)(9) = 0$.	$x^2 - 6x + 11 = 0$ The discriminant is $(-6)^2 - 4(1)(11) = -8$, which is negative.
Number of Solutions	There are two real-number solutions.	There is one real-number solution.	There are no real-number solutions.

Practice.

1. Use the quadratics formula to solve each equation.

$2x^2 + 5x + 3 = 0$

$5x^2 + 16x - 84 = 0$

$4x^2 + 7x - 15 = 0$

$3x^2 - 41x = -110$

$18x^2 - 45x - 50 = 0$

$3x^2 + 44x = -96$

$3x^2 + 19x = 154$

$2x^2 - x - 120 = 0$

$5x^2 - 47x = 156$

$x^2 + 8x + 11 = 0$

$5x^2 + 12x - 2 = 0$

$2x^2 - 16x = -25$

$8x^2 - 7x - 5 = 0$

$6x^2 + 9x = 32$

$3x^2 + 5x = 4$

2. A football player punts a ball. The path of the ball can be modeled by the equation $y = -0.004x^2 + x + 2.5$, where x is the horizontal distance, in feet, the ball travels and y is the height, in feet, of the ball. How far from the football player will the ball land? Round to the nearest tenth of a foot.

3. Which method would you chose to solve each equation. Justify your reasoning. Solve.

$x^2 + 4x - 15 = 0$

$9x^2 - 49 = 0$

$4x^2 - 41x = 73$

$3x^2 - 7x + 3 = 0$

$x^2 + 4x - 60 = 0$

$-4x^2 + 8x + 1 = 0$

4. Find the number of real-number solutions of each equation.

$x^2 - 2x + 3 = 0$

$x^2 + 7x - 5 = 0$

$x^2 + 3x + 11 = 0$

$x^2 - 15 = 0$

$x^2 + 2x = 0$

$9x^2 + 12x + 4 = 0$

5. Solve equations. Use any method.

$3w^2 = 48$

$3x^2 + 2x - 4 = 0$

$6g^2 - 18 = 0$

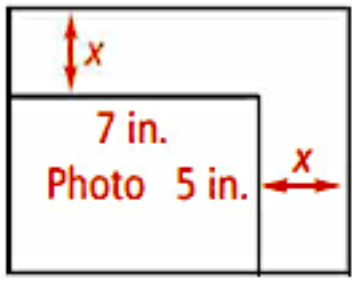
$3p^2 + 4p = 10$

$k^2 - 4k = -4$

$13r^2 - 117 = 0$

6. You operate a dog-walking service. You have 50 customers per week when you charge $14 per walk. For each $1 decrease in your fee for walking a dog, you get 5 more customers per week. Can you ever earn $750 in a week? Explain.
   - What quadratic equation in standard form can you use to model this situation?
   - How can the discriminant of the equation help you solve the problem?

7. Your school wants to take out an ad in the paper congratulating the basketball team on a successful season, as shown below. The area of the photo will be half the area of the entire ad. What is the value of x?



8. Find the discriminant and the solution of each equation in parts (a)-(c).
   a. $x^2 - 6x + 5 = 0$
   b. $x^2 + x - 20 = 0$
   c. $2x^2 - 7x - 3 = 0$
   d. Reasoning When the discriminant is a perfect square, are the solutions rational or irrational? Explain.

9. The solutions of any quadratic equation $ax^2 + bx + c = 0$ are

$$\frac{-b + \sqrt{b^2 - 4ac}}{2a} \text{ and } \frac{-b - \sqrt{b^2 - 4ac}}{2a}.$$

   a) Find a formula for the sum of the solutions.
   b) One solution of $2x^2 + 3x - 104 = 0$ is -8. Use the formula you found in part (a) to find the second solution.

10. For each condition given, tell whether $ax^2 + bx + c = 0$ will always, sometimes, or never have two solutions.
    a) $b^2 < 4ac$
    b) $b^2 = 0$
    c) $ac < 0$