Math 7

Solving Quadratic Equations. Completing the square.

Recall, that we can solve quadratic equations of the form $x^2 = D$ or $(x-h)^2 = D$ by taking square root of both sides of the equation and getting $x = \pm\sqrt{D}$ and $x - h = \pm\sqrt{D}$ (thus $x = \pm\sqrt{D} + h$) respectively.

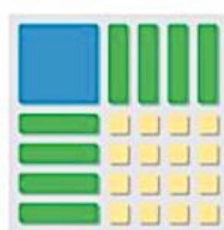
Let see how this can help us solve quadratic equations in their general standard form $ax^2 + bx + c = 0$

Let first see how we can use graphic (or geometric) representation of quadratic expressions to see how they can be changed into a prefect squares.

You can use algebra tiles to model changing an expression into a perfect square. The algebra tiles here represent the expression $x^2 + 8x$.

You can represent the same expression by rearranging the tiles to form part of a square. Notice that the x-tiles have been split evenly into two groups of four.

You can complete the square by adding 4^2, or 16, 1-tiles. The completed square is $x^2 + 8x + 16$ or $(x+4)^2$.



In general, you can change the expression $x^2 + bx$ into a perfect-square trinomial by adding $\left(\frac{b}{2}\right)^2$ to $x^2 + bx$. Tis process is called **completing the square**. The process is the same whether b is positive or negative.

Example 1. What is the value of c such that $x^2 - 16x + c$ is a perfect square?

The value of b is -16. The term to add to $x^2 - 16x$ is $\left(-\frac{16}{2}\right)^2$, or 64. So, $c = 64$.

In general, to solve the equation in the form $x^2 + bx + c = 0$, first subtract constant term 0 from each side of the equation.

Example 2. Solve $x^2 - 14x + 16 = 0$.

$$x^2 - 14x + 16 = 0$$
$$x^2 - 14x = -16$$

Now, we add $\left(-\frac{14}{2}\right)^2 = 49$ to each side $x^2 - 14x + 49 = 16 + 49$

$$(x-7)^2 = 33$$
$$x - 7 = \pm\sqrt{33}$$
$$x = \pm\sqrt{33} + 7$$

*In case if $a \neq 0$ in $ax^2 + bx + c = 0$ we first divide both side of the equations by a and then proceed as described above.

Practice.

1. Find the value of c such that each expression is a perfect square trinomial.

$x^2 + 18x + c$

$z^2 + 22z + c$

$p^2 - 30p + c$

$k^2 - 5k + c$

$g^2 + 17g + c$

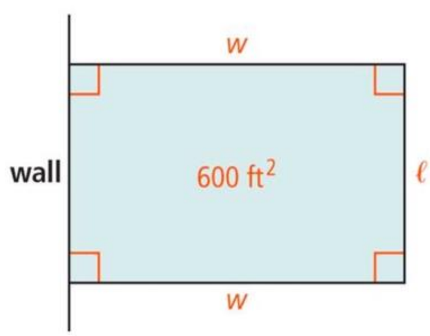
$q^2 - 4q + c$

2. Solve each equation by comleting the square.

$g^2 + 7g = 144$	$4a^2 - 8a = 24$
$r^2 - 4r = 30$	$2y^2 - 8y - 10 = 0$
$m^2 + 16m = -59$	$5n^2 - 3n - 15 = 10$
$a^2 - 2a - 35 = 0$	$4w^2 + 12w - 44 = 0$
$m^2 + 12m + 19 = 0$	$3r^2 + 18r = 21$
$w^2 - 14w + 13 = 0$	$2v^2 - 10v - 20 = 8$

3. A park is installing a rectangular reflecting pool surrounded by a concrete walkway of uniform width. The reflecting pool will measure 42 ft by 26 ft. There is enough concrete to cover 460 ft^2 for the walkway. What is the maximum width x of the walkway?
   - How can drawing a diagram help you solve this problem?
   - How can you write an expression in terms of x for the area of the walkway?

4. A school is fencing in a rectangular area for a playground. It plans to enclose the playground using fencing on three sides, as shown below. The school has budgeted enough money for 75 ft of fencing material and would like to make a playground with an area of 600 ft^2.



   a) Let w represent the width of the playground. Write an expression in terms of w for the length of the playground.
   b) Write and solve an equation to find the width w. Round to the nearest tenth of a foot.
   c) What should the length of the playground be?

5. Solve each equation. If there is no real number solution, write *no solution*.

$q^2 + 3q + 1 = 0$

$s^2 + 5s = -11$

$w^2 + 7w - 40 = 0$

$z^2 - 8z = -13$

$4p^2 - 40p + 56 = 0$

$m^2 + 4m + 13 = -8$

$2p^2 - 15p + 8 = 43$

$3r^2 - 27r = 3$

$s^2 + 9s + 20 = 0$

6. A classmate was completing the square to solve $4x^2 + 10x = 8$. For her first step she wrote $4x^2 + 10x + 25 = 8 + 25$. What was her error?

7. Explain why completing the square is a better strategy for solving $x^2 - 7x - 9 = 0$ than graphing or factoring.