Math 7 Factoring simple quadratic expressions $x^2 + bx + c$

You can write some expression of the form $x^2 + bx + c$ as a product $(x + x_1)(x + x_2)$.

Now, let see how this works:

$$(x+3)(x+7) = x^2 + 7x + 3x + 7 \cdot 3$$
$$= x^2 + (7+3)x + 7 \cdot 3$$
$$= x^2 + 10x + 21$$

coefficient is 1 | sum of 7 and 3 | product of 7 and 3

Analogously,

$$(x + x_1)(x + x_2) = x^2 + x_1x + x_2x + x_1x_2 = x^2 + (\boldsymbol{x_1 + x_2})x + \boldsymbol{x_1x_2}$$

Compare to $x^2 + \boldsymbol{b}x + \boldsymbol{c}$

Here, $\boldsymbol{(x_1 + x_2) = b}$, $\boldsymbol{x_1x_2 = c}$

Examples:

1. What is the factored form of $x^2 + 8x + 15$?

   We first list all factors of the constant term 15:

15	
± 1	± 15
± 3	± 5

We then identify the pair that has a sum of 8: **3 and 5**.

$$x^2 + 8x + 15 = (x + 3)(x + 5)$$

Let's check

$$(x + 3)(x + 5) = x^2 + 5x + 3x + 15 = x^2 + 8x + 15$$

2. What is the factored form of $x^2 - 11x + 24$?

We first list all factors of the constant term 24:

24	
± 1	± 24
± 2	± 12
± 3	± 8
± 4	± 6

We then identify the pair that has a sum of -11: **-3 and -8**.

$$x^2 - 11x + 24 = (x - 3)(x - 8)$$

Let's check

$$(x - 3)(x - 5) = x^2 - 8x - 3x + 24 = x^2 - 11x + 24$$

3. What is the factored form of $x^2 + 2x - 15$?

We first list all factors of the constant term 15:

-15	
± 1	∓ 15
± 3	∓ 5

We then identify the pair that has a sum of 8: -**3 and 5**.

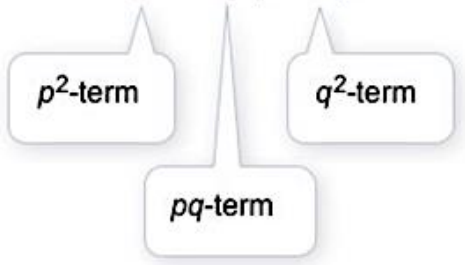
$$x^2 + 8x + 15 = (x - 3)(x + 5)$$

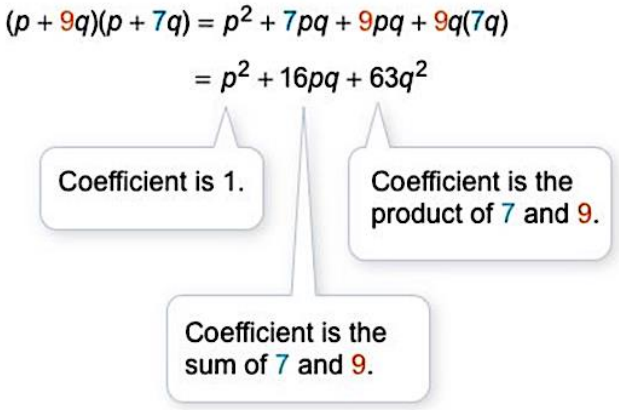
Let's check

$$(x - 3)(x + 5) = x^2 + 5x - 3x - 15 = x^2 + 8x - 15$$

Same idea is used to factor quadratic expressions that have more than one variable. Consider the following example:

$$(p + 9q)(p + 7q) = p^2 + 7pq + 9pq + 9q(7q)$$
$$= p^2 + 16pq + 63q^2$$





This suggests that a trinomial with two variables may be factorable if the first term includes the square of one variable, the middle term includes both variables, and the last term includes the square of the other variable.

Examples:

1. What is the factored form of $x^2 + 6xy - 55y^2$?

   We first list all factors of -55:

-55	
± 1	∓ 55
± 11	∓ 5

   We then identify the pair that has a sum of 6: **-5 and 11**.

$$x^2 + 6xy - 55y^2 = (x - 5y)(x + 11y)$$

Exercises

1. Factor:
   a) $x^2 + 7x + 12$
   b) $r^2 13r + 42$
   c) $p^2 + 3p - 40$
   d) $a^2 + 12ab + 32b^2$

2. The area of a rectangle is equal to $n^2 - 3n - 28$. What are the possible dimensions of the rectangle?

3. Factor:
   a) $k^2 + 5k + 6$
   b) $x^2 - 7x + 10$
   c) $t^2 - 10t + 24$
   d) $v^2 + 12v + 20$
   e) $y^2 + 6y + 5$
   f) $t^2 + 10t + 16$
   g) $x^2 + 15x + 56$
   h) $n^2 - 15n + 56$
   i) $r^2 - 11r + 24$
   j) $q^2 - 8q + 12$
   k) $q^2 + 3q - 54$
   l) $z^2 - 2z - 48$
   m) $n^2 - 5n - 50$
   n) $y^2 + 8y - 9$
   o) $r^2 + 6r - 27$
   p) $w^2 - 7w - 8$
   q) $z^2 + 2z - 8$
   r) $x^2 + 5x - 6$
   s) $v^2 + 5v - 36$
   t) $n^2 - 3n - 10$

4. The area of a rectangular desk is $d^2 - 7d - 18$. What are the possible dimensions of the desk?

5. The area of a rectangular rug is $r^2 - 3r - 4$. What are the possible dimensions of the rug?

6. Factor:

   a) $k^2 + 5kn - 84n^2$
   b) $p^2 - 8pq - 33q^2$
   c) $x^2 - 16xy + 48y^2$
   d) $r^2 + 19rs + 90s^2$
   e) $g^2 - 12gh + 35h^2$
   f) $m^2 - 3mn - 28n^2$
   g) $x^2 + 3xy - 18y^2$
   h) $w^2 - 14wz + 40z^2$
   i) $p^2 + 11pq + 24q^2$

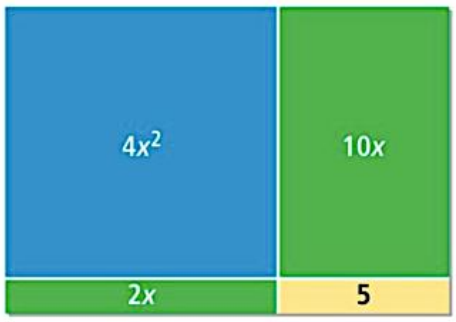
7. Suppose you can factor $x^2 + bx + c$ as $(x + p)(x + q)$

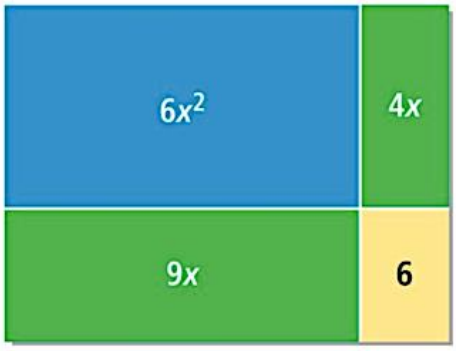
a) Explain what so you know about p and q when $c > 0$?
b) Explain what so you know about p and q when $c < 0$?

8. The area of a parallelogram is given by the expression $x^2 - 14x + 24$. The base of the parallelogram is $x - 2$. What will be an expression for the height of the parallelogram?
    - What is the formula for the area of the parallelogram?
    - How can you tell whether the expression that represents the height has a positive or negative constant term?

9. A rectangular skateboard park has an area of $x^2 + 15x + 54$. What are the possible dimensions of the park?

10. Write a quadratic expression for every case modeled below. Then factor each expression.





11. Let $x^2 - 13x - 30 = (x + p)(x + q)$
    a) What do you know about the signs of p and q?
    b) Suppose $|p| > |q|$. Which number, p or q, is a negative integer?

12. Factor
    a) $x^2 + 27x + 50$
    b) $g^2 - 18g + 45$
    c) $k^2 - 18k - 63$
    d) $d^2 + 30d - 64$
    e) $s^2 - 10st - 75t^2$
    f) $h^2 + 9hj - 90j^2$

Challenge yourself

13. Factor each expression
Example: $\boldsymbol{n^6 + n^3 - 42 = (n^3)^2 + n^3 - 42 = (n^3 - 6)(n^3 + 7)}$

a) $x^{12} + 12x^6 + 35$
b) $t^8 + 5t^4 - 24$
c) $r^6 - 21r^3 + 80$
d) $m^{10} + 18m^5 + 17$
e) $x^{12} - 19x^6 - 120$
f) $p^6 + 14p^3 - 72$