MATH 7
ASSIGNMENT 15: EQUATIONS OF THE LINE AND THE CIRCLE

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Coordinates

After we choose an origin (usually denoted O) and two perpendicular axes, every point in the plane is described by a pair of numbers, its x and y coordinates. We will write (a, b) for point with x coordinate a and y-coordinate b.

Distance between two points is given by

$$d((x_1, y_1), (x_2, y_2)) = \sqrt{(x_1 - x_2)^2 + (y_1 - y_2)^2}$$

Equation of the Line

A general equation of non-vertical line is $y = mx + b$; the number m is called the *slope* of this line. It can also be defined as follows: if (x_0, y_0) and (x_1, y_1) are two points on this line, then $\frac{y_1 - y_0}{x_1 - x_0} = m$.

Another common form of writing the equation of a line is $ax + by = c$.

Equation of the Circle

Equation of a circle with center at (x_0, y_0) and radius r is $(x - x_0)^2 + (y - y_0)^2 = r^2$.

Homework

1. Find the equation of a line going through point $(5, 7)$ and having slope 2.

2. Find the equation of a line through two points, $(3, 4)$ and $(5, 7)$.

3. What is the equation of a circle centered in $O(-3, 1)$ and of radius 2.

4. What are the coordinates of the center and the radius of the circle defined by $(x + 7)^2 + (y - 3)^2 = 2$?

5. Show that $(3, 5)$ is equidistant from $(-1, 2)$ and $(3, 0)$. (*Equidistant* means that the distances are the same)

6. Let $A = (3, 5)$, $B = (6, 1)$ be two of the vertices of a square $ABCD$ (the vertices are labeled A, B, C, D going counterclockwise). Find the coordinates of points C, D and of the center of the square. Find the area of this square.

Extra Problems (Optional)

1. Show that two lines are parallel if and only if they have the same slope.

2. (a) Show that 90° counterclockwise rotation sends point $(2, 1)$ to point $(-1, 2)$. Where would it send point (x, y)?

(b) Show that two lines are perpendicular if and only if their slopes are related by $m_1 = -1/m_2$.

3. Let C be the circle with center at $(0, 1)$ and radius 2, and l - the line with slope 1 going through the origin. Find the intersection points of the circle C and line l, and compute the distance between them.

4. Prove the following formula for the distance from a point to the line: the distance from point $P = (u, v)$ to the line given by equation $ax + by = 0$ is

$$d = \frac{|au + bv|}{\sqrt{a^2 + b^2}}$$

5. Prove that the set of all points P satisfying the following equation

$$\text{distance from } P \text{ to the origin} = 2 \cdot (\text{distance from } P \text{ to } (0, 3))$$

is a circle. Find its radius and center.