MATH 7
ASSIGNMENT 13: MORE PROBLEMS IN COMBINATORICS

JAN 19, 2020

Today we will practice more combinatorics problems. Recall the basic idea:

Principle of Counting

In how many ways can you arrange N objects in a sequence (from the 1st to the Nth)?

There are N ways to choose the first one, $N-1$ ways to choose the second object once the first one has been chosen, $N-2$ ways to choose the third object once the first and the second objects have been chosen, an so one. So the Answer is $N!$, which is defined by

$$N! := N(N-1)\dots 1.$$

This is also called P_N, the number of permutations of N elements.

We will see that we can generalize this idea to solve many problems of combinatorics. This idea is sometimes called the *fundamental principle of counting.*

We can generalize this idea in many ways: in some problems we want to count possible subsets, or possible subsequences, etc. The best way is to practice with many different problems.

Homework

1. How many words one can get by permuting letters of the word "tiger"? of the word "rabbit"? of the word "mammoth"?

2. Fred has 37 favorite recipes in his recipe book.
  (a) How many different menus can he prepare for the day (breakfast, luch and dinner)?
  (b) What if 12 of these recipes are breakfast recipes?
  (c) If Fred is asked to recommend 3 recipes, in how many way can he choose them?

3. 5 friends go to a ramen restaurant for dinner. In this restaurant there are 17 different types of ramen.
  (a) If each one orders one ramen, how many possible combinations are there?
  (b) If each one orders one ramen so that they all choose differently, how many possible combinations are there?
  (c) In how many ways can one choose 5 types of ramen (for oneself) from the menu?

Extra Problems (Optional)

1. A staircase consists of 7 steps not counting upper and lower landings. Going downstairs one can jump over some of steps (even all seven). In how many ways one can descend this staircase?

2. A monomial is a product of powers of variables, i.e. an expression like x3y7.
  (a) How many monomials in variables x,y of total degree of exactly 15 are there? (Note: this includes monomials which only use one of the letters, e.g. x15.)
  (b) Same question about monomials in variables x,y,z.[Hint: if you write 15 letters in a row, you need to indicate where x's end and y's begin — you can insert some kind of marker to indicate where it happens.]
  (c) How many monomials in variables x, y of degree at most 15 are there?
  (d) How many monomials in variables x, y, z of degree at most 15 are there?

3. In how many ways can seven friends seat around a round table?

4. Given 6 different colors, in how many ways can we paint a cube? Each face should have a different color, and two configurations which are equivalent by a rotation of the cube are considered the same.