MATH 7
ASSIGNMENT 9: THE TRIGONOMETRIC CIRCLE
NOV 17, 2019

Radians

Until now, we have been measuring angles in degrees, which are defined by saying that a full turn corresponds to $360°$.

An alternative way to measure angles is by radians, which are defined in the following way: given an angle α, it's measure in radians is the ratio of an arc of circumference with angle α by the radius of the circumference.

For example, the angle $360°$ corresponds to a full circle. Since the perimeter of a circle is $2\pi R$, dividing by R gives:

$$360° \leftrightarrow 2\pi \text{ rad}.$$

In the same way, half a circle corresponds to an angle of π radians. By similar arguments, we can translate all the angles that appeared in our previous table:

Trigonometric Functions							
Function	Notation	Definition	0	$\frac{\pi}{6}$	$\frac{\pi}{4}$	$\frac{\pi}{3}$	$\frac{\pi}{2}$
sine	$\sin(\alpha)$	$\frac{\text{opposite side}}{\text{hypotenuse}}$	0	$\frac{1}{2}$	$\frac{\sqrt{2}}{2}$	$\frac{\sqrt{3}}{2}$	1
cosine	$\cos(\alpha)$	$\frac{\text{adjacent side}}{\text{hypotenuse}}$	1	$\frac{\sqrt{3}}{2}$	$\frac{\sqrt{2}}{2}$	$\frac{1}{2}$	0
tangent	$\tan(\alpha)$	$\frac{\text{opposite side}}{\text{adjacent side}}$	0	$\frac{1}{\sqrt{3}}$	1	$\sqrt{3}$	∞

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Trigonometric Circle

A very useful tool in understanding the trigonometric functions is the *trigonometric circle* (see figure below): in order to find the sine and cosine of a positive angle α, we just have to "walk" around the circle a distance α, starting from the point $(1,0)$ in anti clockwise direction. Then the coordinates of the point we arrive at are $(\cos\alpha, \sin\alpha)$. For α negative, we define the sine and cosine in the same way, but walking in the clockwise direction.

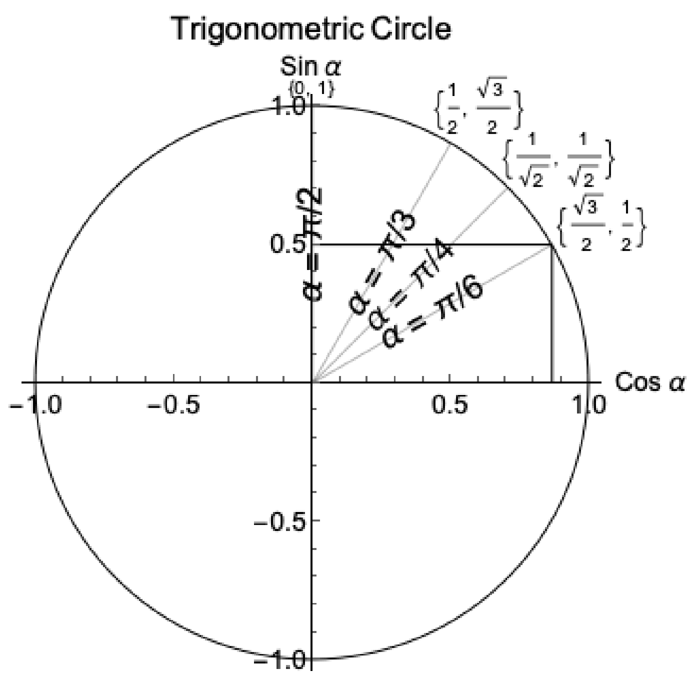

FIGURE 1. Trigonometric circle: in order to find the sine and cosine of angle α, we just have to "walk" around the circle a distance α, starting from the point $(1,0)$. Then the coordinates of the point we arrive at are $(\cos\alpha, \sin\alpha)$.

Graph of the Function Sin (x)

By looking at the values of sine as we go around the trigonometric circle, we find out a few facts like:

- $\sin 0 = \sin \pi = 0$
- $\sin x$ increases from 0 to $\frac{\pi}{2}$.
- At $x = \frac{\pi}{2}$, $\sin x$ reaches it's maximum value, 1.
- At $x = \frac{3\pi}{2}$, $\sin x$ reaches it's minimum value, -1.
- $\sin x + 2\pi = \sin x$.

We can see all of these facts clearly in the graph of the function $\sin x$:

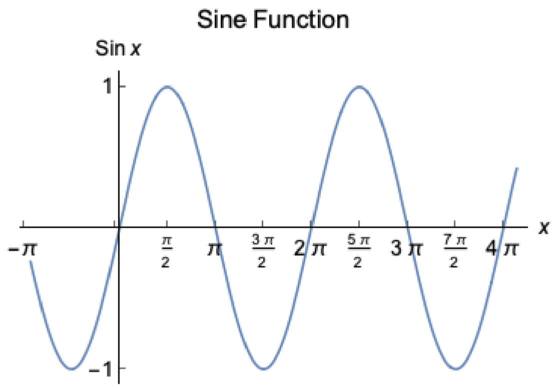

FIGURE 2. Graph of Sine.

Homework

1. Draw a large trigonometric circle. Then, remembering that 2π corresponds to a full circle, find the points corresponding to (write the corresponding letter on the correct point)

(a) π
(b) $\frac{3\pi}{2}$
(c) $\frac{3\pi}{4}$
(d) $-\frac{5\pi}{4}$
(e) 11π
(f) -3π
(g) $\frac{25\pi}{3}$
(h) $-\frac{19\pi}{6}$

2. Now use your trigonometric circle and figure 1 to complete this table:

Point	Sine	Cosine
(a)	0	-1
(b)		
(c)		
(d)		
(e)		
(f)		
(g)		
(h)		

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3. Using the trigonometric circle, check where appropriate:

x	$\sin x \geq \sqrt{3}/2$	$1/2 < \sin x < \sqrt{3}/2$	$-\sqrt{2}/2 < \sin x \leq 1/2$	$\sin x \leq -\sqrt{2}/2$
$\pi/7$			✓	
$2\pi/7$				
$-3\pi/5$				
$5\pi/8$				
$25\pi/9$				

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4. Using the trigonometric circle, show that $\cos x = \sin(x + \pi/2)$ for any angle x. Then use this fact and the graph of the Sine function (figure 2) to construct (draw) the graph of the Cosine function.

5. Find all real numbers x such that $(\sin x)^2 = 3/4$