

**MATH 7**  
**ASSIGNMENT 8: LAW OF SINES**  
NOV 17, 2019

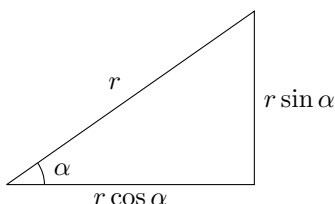
**Trigonometric Ratios**

First recall the definitions of the trigonometric ratios:

$$\begin{aligned}\sin \alpha &= \frac{\text{opposite side}}{\text{hypotenuse}} \\ \cos \alpha &= \frac{\text{adjacent side}}{\text{hypotenuse}} \\ \tan(\alpha) &= \frac{\text{opposite side}}{\text{adjacent side}}\end{aligned}$$

for any given right triangle.

A very useful consequence is that the sides of such a triangle can all be written in terms of the length of the hypotenuse and trigonometric ratios in the standard form



**Trigonometric Identities**

The trigonometric ratios are related by various *identities*. The most prominent trigonometric identity is

$$\sin^2(\alpha) + \cos^2(\alpha) = 1.$$

This fact is automatically satisfied, for any angle  $\alpha$ . This is why we call this an identity instead of an equation. Let us try to derive it:

- A right triangle with hypotenuse  $c$  and an angle  $\alpha$  is given. Express the remaining 2 sides ( $a$  and  $b$ ) of triangle using only  $c$  and  $\alpha$ .
- Using expressions obtained for  $a$  and  $b$ , express the hypotenuse  $c$  and simplify.

**Law of Sines**

Given a triangle  $\triangle ABC$  with sides  $a$ ,  $b$ , and  $c$ , the following is always true:

$$\frac{a}{\sin(A)} = \frac{b}{\sin(B)} = \frac{c}{\sin(C)}.$$

**Proof:** To see why the Law of Sines is true, refer to the Figure 1. The height of the triangle  $h = b \sin C$ , and therefore the area of the triangle is  $S = \frac{1}{2}ab \sin C$ . Similarly,  $S = \frac{1}{2}bc \sin A = \frac{1}{2}ac \sin B$ . Thus,

$$bc \sin A = ac \sin B = ab \sin C$$

Dividing by  $abc$ , we get:

$$\frac{\sin A}{a} = \frac{\sin B}{b} = \frac{\sin C}{c}.$$

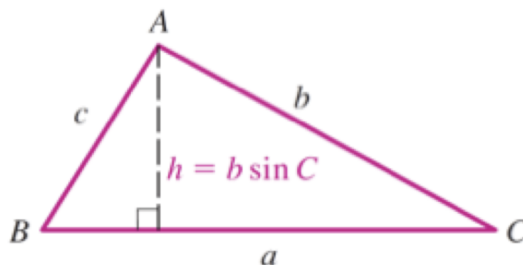


FIGURE 1. Law of Sines

### Homework

1. In a triangle  $\triangle ABC$ , we have  $\angle A = 40^\circ$ ,  $\angle B = 60^\circ$ , and  $AB = 2$  cm. What is the remaining angle and side lengths? (Hint: Use Law of sines)
2. In an isosceles triangle, the angle between the equal sides is equal to  $30^\circ$ , and the height is 8. Find the sides of the triangle.
3. A right triangle  $\triangle ABC$  is positioned such that  $A$  is at the origin,  $B$  is in the 1st quadrant ( $B_x > 0$  and  $B_y > 0$ ) and  $C$  is on the positive horizontal axis ( $C_x > 0$  and  $C_y = 0$ ). If length of side  $AB$  is 1, and  $AB$  makes a  $35^\circ$  angle with positive  $x$  axis, what are the coordinates of  $B$ ?
4. Consider a parallelogram  $ABCD$  with  $AB = 10$ ,  $AD = 4$  and  $\angle BAD = 50^\circ$ . Find the length of diagonal  $AC$ .
5. A regular heptagon (7 sides) is inscribed into a circle of radius 1.
  - (a) What is the perimeter of the heptagon?
  - (b) What is the area of the heptagon?
6. In the trapezoid below,  $AD = 5$  cm,  $AB = 2$  cm, and  $\angle A = \angle D = 70^\circ$ . Find the length  $BC$  and the diagonals. [You can use:  $\sin(70^\circ) \approx 0.94$ ,  $\cos(70^\circ) \approx 0.34$ .]
7. To determine the distance to the enemy gun (point  $E$  in the figure below), the army unit placed two observers (points  $A, B$  in the figure below) and asked each of them to measure the angles using a special instrument. The results of the measurements are shown below. If it is known that the distance between the observers is 400 meters, can you determine how far away from observer  $A$  is the enemy gun?

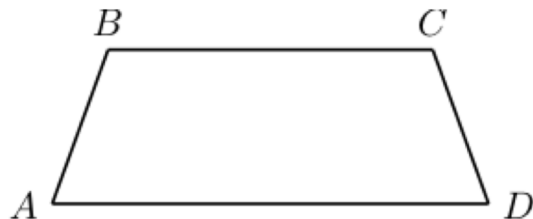


FIGURE 2. Problem 6

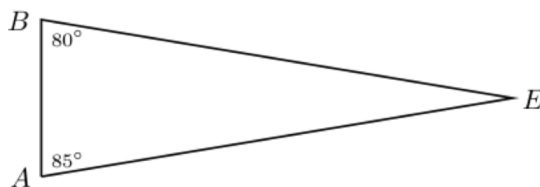


FIGURE 3. Problem 7