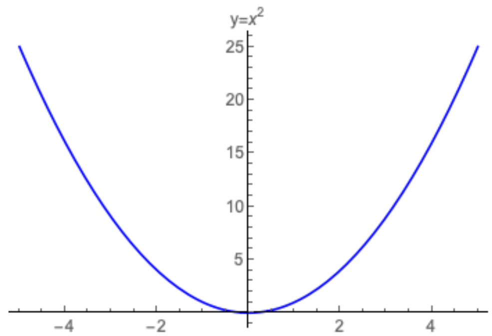
MATH 7
ASSIGNMENT 5: GRAPH OF A QUADRATIC POLYNOMIAL: PARABOLA

OCT 20, 2019

1. The Parabola

Today we will practice how to solve quadratic equations and inequalities, and see how they relate to the graph of quadratic polynomials: the parabola.

The shape of the curve $y = ax^2 + bx + c$ is always that of a *parabola*. We will see later a more geometrical definition of the parabola, but for now it suffices to have an idea of how it looks like. For example, the curve $y = x^2$ is



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Other quadratic polynomials look similar. Let us use what we learned about them:

2. Quadratic Equations: Summary

- A **quadratic polynomial** is an expression of the form $p(x) = ax^2 + bx + c$.
- **Roots** of a quadratic polynomial are numbers such that $p(x) = 0$. If x_1, x_2 are roots, then $p(x) = a(x - x_1)(x - x_2)$.
- **Vietá formulas**: If x_1, x_2 are roots of $ax^2 + bx + c$, then

$$x_1 + x_2 = -\frac{b}{a} \tag{1}$$

$$x_1 x_2 = \frac{c}{a} \tag{2}$$

- **Completing the square**: we can rewrite

$$ax^2 + bx + c = a\left(x + \frac{b}{2a}\right)^2 - \frac{D}{4a} = a\left(\left(x + \frac{b}{2a}\right)^2 - \frac{D}{4a^2}\right) \tag{3}$$

  where $D = b^2 - 4ac$.

  From this, one gets the **quadratic formula**: if $D < 0$, there are no roots; if $D \geq 0$, then the roots are

$$x_{1,2} = \frac{-b \pm \sqrt{D}}{2a} \tag{4}$$

- From formula (3), we see that:
  - If $a > 0$, then the **smallest** possible value of $p(x)$ is $-\frac{D}{4a}$, which happens when $x = -\frac{b}{2a}$. In this case the graph is a parabola with branches going up.
  - If $a < 0$, then the **largest** possible value of $p(x)$ is $-\frac{D}{4a}$, which happens when $x = -\frac{b}{2a}$. In this case the graph is a parabola with branches going down.

– If $D < 0$, the parabola does not cross the x axis, while if $D > 0$ the parabola crosses the x axis at x_1 and x_2 given by (4)

Homework

1. In each case, solve the equation, then the inequalities, and then sketch the graph of the parabola.
(a) $x^2 - 5x + 5 = 0$, $x^2 - 5x + 5 > 0$, $x^2 - 5x + 5 < 0$, sketch the graph $y = x^2 - 5x + 5$
(b) $x^2 - 5x - 14 = 0$, $x^2 - 5x - 14 > 0$, $x^2 - 5x - 14 < 0$, sketch the graph $y = x^2 - 5x - 14$
(c) $-x^2 + 11x - 28 = 0$, $-x^2 + 11x - 28 > 0$, $-x^2 + 11x - 28 < 0$, sketch the graph $y = -x^2 + 11x - 28$
(d) $-6x^2 - 19x + 7 = 0$, $-6x^2 - 19x + 7 > 0$, $-6x^2 - 19x + 7 < 0$, sketch the graph $y = -6x^2 - 19x + 7$
(e) $x^2 - x - 1 = 0$, $x^2 - x - 1 > 0$, $x^2 - x - 1 < 0$, sketch the graph $y = x^2 - x - 1$
(f) $-x^2 + 2x + 2 = 0$, $-x^2 + 2x + 2 > 0$, $-x^2 + 2x + 2 < 0$, sketch the graph $y = -x^2 + 2x + 2$
(g) $x^2 + 2x - 3 = 0$, $x^2 + 2x - 3 > 0$, $x^2 + 2x - 3 < 0$, sketch the graph $y = x^2 + 2x - 3$
(h) $x^2 + 2x + 3 = 0$, $x^2 + 2x + 3 \geq 0$, $x^2 + 2x + 3 < 0$, sketch the graph $y = x^2 + 2x + 3$
(i) $-x^2 + 6x - 9 = 0$, $-x^2 + 6x - 9 \geq 0$, $-x^2 + 6x - 9 < 0$, sketch the graph $y = -x^2 + 6x - 9$
(j) $3x^2 + x - 1 = 0$, $3x^2 + x - 1 \geq 0$, $3x^2 + x - 1 \leq 0$, sketch the graph $y = 3x^2 + x - 1$

2. For what values of a does the polynomial $x^2 + ax + 14$ has no roots? exactly one root? two roots?

3. Let x_1, x_2 be the roots of the equation $x^2 + 3x + 4 = 0$. Without calculating the roots, find:
(a) $x_1^2 + x_2^2$
(b) $\frac{1}{x_1^2} + \frac{1}{x_2^2}$

4. The sum of reciprocals of two consecutive integers is $13/42$. Find the integers. What are the consecutive integers for which the sum of their reciprocals is larger than $13/42$? Less than $13/42$?

5. Of all the rectangles with perimeter 4, which one has the largest area?
[**Hint:** if sides of the rectangle are a and b, then the area is $A = ab$, and the perimeter is $2a + 2b = 4$. Thus, $b = 2 - a$, so one can write A using only a...]

***6.** What is the value of

$$x = \sqrt{2 + (\sqrt{2 + (\sqrt{2 + (\sqrt{2 + (....}}}}$$

[**Hint:** Calculate x^2.]

***7.** A triangle ABC, has corners $A(-3, 0)$, $B(0, 3)$ and $(3, 0)$. The line $y = \dfrac{1}{3}x + 1$ separates the triangle in 2. What is the area of the piece lying below the line?