# MATH 7
# ASSIGNMENT 3: LINEAR AND QUADRATIC EQUATIONS

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The simplest types of algebraic equations are linear and quadratic equations. You will review/practice solving linear equations and systems of linear equations in your homework. We also introduce a first method to solve quadratic equations: using Vieta's formulas.

## 1. **Vieta formulas**

If an equation $p(x) = 0$ has root $a$ (i.e., if $p(a) = 0$), then $p(x)$ is divisible by $(x-a)$, i.e. $p(x) = (x-a)q(x)$ for some polynomial $q(x)$. In particular, if $x_1$; $x_2$ are roots of quadratic equation $ax^2 + bx + c = 0$, then $ax^2 + bx + c = a(x - x_1)(x - x_2)$.

Therefore, if $a = 1$, then

$$\begin{aligned} x_1 + x_2 &= -b \\ x_1 x_2 &= c \end{aligned}$$

These formulas are called *Vieta Formulas.*

## Homework

**1.** Solve the following equations:
(a) $\frac{x+3}{x+1} = 4$
(b) $2x + 25 = 5x + 10$
(c) $\frac{x}{2} + 1 = \frac{4x}{7}$
(d) $x = \frac{x}{4} + 6$
(e) $x + 2(x - 5) = \frac{1}{2}(x + 3)$

**2.** The pet store sells parrots and canaries. A canary costs twice as much as a parrot. One customer bought 5 canaries and 3 parrots, while the other bought 3 canaries and 5 parrots. One of the customers paid \$20 more than the other. How much does each bird cost?

**3.** The teacher asked the students to multiply a given number by 4 and then add 15. However, one of the students multiplied the number by 15 and then added 4 — and still got the correct answer. What number was it?

**4.** Solve the following systems of equations

(a)

$$\begin{aligned} x &= 5 \\ 20x + 5y &= 100 \end{aligned}$$

(b)

$$\begin{aligned} -8x + y &= -4 \\ -21x + 2y &= -13 \end{aligned}$$

(c)

$$\begin{aligned} 7x - 3y &= 27 \\ 5x - 6y &= 0 \end{aligned}$$

(d)

$$\begin{aligned} 2(x-2) - 3(x+y) &= 3 \\ (x+1)(y-2) &= xy - 9 \end{aligned}$$

(e)

$$\frac{2x-1}{5} + \frac{3y-2}{4} = 2$$
$$\frac{3x+1}{5} - \frac{3y+2}{4} = 0$$

**5.** Let $a$ and $b$ be some numbers. Use the formulas discussed in previous classes to express the following expressions using only $(a+b) = x$ and $ab = y$.
**Example:** Let's express $a^2 + b^2$ using only $a+b$ and $ab$. We know that $(a+b)^2 = a^2 + 2ab + b^2$. From here, we get:

$$a^2 + b^2 = (a+b)^2 - 2 \times ab = x^2 - 2 \times y$$

(a) $(a-b)^2$
(b) $\frac{1}{a} + \frac{1}{b}$
(c) $a - b$
(d) $a^2 - b^2$
(e) $a^3 + b^3$ (Hint: first compute $(a+b)(a^2+b^2)$)

**6.** Let $x_1, x_2$ be roots of the equation $x^2 + 5x - 7 = 0$. Find
(a) $x_1^2 + x_2^2$
(b) $(x_1 - x_2)^2$
(c) $\frac{1}{x_1} + \frac{1}{x_2}$
(d) $x_1^3 + x_2^3$

**7.** Solve the following equations:
(a) $x^2 - 5x + 6 = 0$
(b) $x^2 = 1 + x$
(c) $\sqrt{2x+1} = x$
(d) $x + \frac{1}{x} = 3$

**8.** Solve the equation $x^4 - 3x^2 + 2 = 0$

**9.** (a) Prove that for any $a > 0$, we have $a + \frac{1}{a} \geq 2$, with equality only when $a = 1$.
(b) Show that for any $a, b \geq 0$, one has $\frac{a+b}{2} \geq \sqrt{ab}$. (The left hand side is usually called the *arithmetic mean* of $a, b$; the right hand side is called the *geometric mean* of $a, b$.)