MATH 7
ASSIGNMENT 2: MANIPULATING ALGEBRAIC EXPRESSIONS

SEP 21, 2019

Today we will review and also practice various rules on how to manipulate algebraic expressions. We need to be sharp on those before we move on to algebraic equations!

1. Exponents Laws

If a is a real number, n is a positive integer, $a^n = \underbrace{a \times a \times \cdots \times a}_{n\text{-times}}$. We define $a^0 = 1$ and $a^{-n} = \frac{1}{a^n}$. Then the following properties follow:

- $a^m \times a^n = a^{m+n}$
- $a^m \div a^n = a^{m-n}$
- $(ab)^n = a^n b^n$
- $\left(\frac{a}{b}\right)^n = \frac{a^n}{b^n}$
- $(a^m)^n = a^{mn}$

2. Radicals

- $a^{\frac{m}{n}} = \sqrt[n]{a^m}, n \neq 0$
- $\sqrt{ab} = \sqrt{a}\sqrt{b}$

3. Main Algebraic Identities

- $(a+b)^2 = a^2 + 2ab + b^2$
- $(a-b)^2 = a^2 - 2ab + b^2$
- $a^2 - b^2 = (a-b)(a+b)$

Homework

Required Problems. These problems should be handed in by every student in the class.

1. The difference between two numbers is $\frac{5}{12}$. If $\frac{3}{4}$ of the larger number is $\frac{3}{8}$ more than $\frac{1}{2}$ of the smaller, find the larger number.

2. Without a calculator, compute

$$19999 \cdot 20001$$

Is there a shorter way of doing it than the straightforward multiplication?

3. Expand

(a) $2x(a + 2b + 3c)$
(b) $-3y(a - ay + by)$
(c) $(a^2 + 2a + 1)(a + 1)$
(d) $(b^2 - 2b + 1)(b - 1)$
(e) $(4x - 7y)(4x + 7y)$
(f) $(6x^2 - y)(7x^2 - 2x - 5)$

4. Factor (i.e., write as a product) the following expressions:

(a) $ac + ab$
(b) $x^2 + 3x^3$
(c) $x^2 - 2x - yx + 2y$
(d) $4x^2 - 4x + 1$
(e) $4x^2 + 16x + 2xy + 8y$

(f) $x^2(x+4)+5(x+4)$
(g) $100x^8y^2-16x^4y^6$
(h) $a^2+4ab+4b^2$
(i) a^2-2a+1
(j) x^2-7 [Hint: $7=(\sqrt{7})^2$.]
(k) a^4-b^4 [Hint: $a^4=(a^2)^2$.]

5. John takes 15 min to walk from school to the bus station. Jim takes 20 min to walk from the school to the bus station. If the difference in their speeds is 2 km/h, how far is the station from the school?

6. Simplify:

$$\text{(a) } \frac{1}{x+1}-\frac{1}{x-1} \qquad \text{(b) } \left(1+\frac{1}{x}\right)\div(x+1) \qquad \text{(c) } \left(1+\frac{1}{x}\right)\div\left(1-\frac{1}{x}\right)$$

7. Simplify:
(a) $\sqrt{\frac{56}{13}}\cdot\sqrt{\frac{26}{7}}=$
(b) $\sqrt{48}=$
(c) $\frac{\sqrt{48}}{\sqrt{15}}=$

8. Express the following expressions in the form $2^r3^sa^mb^n$:
(a) $8a^3b^2(27a^3)(2^5ab)=$
(b) $3^2(2ab)^3(16a^2b^5)(24b^2a)=$
(c) $16a^2b^3(6ab^4)(ab^2)^3=$

9. Expand as sums of powers of x:
(a) $(2x+5)^2=$
(b) $(2-4x)^2=$
(c) $(1-2x)^2=$
(d) $(1-x)^2(2-x)=$
(e) $(2x+1)^2(2-3x)=$

Additional Problems. If you have finished the above, here are some additional problems for you. You can hand in all of them together.

1. Factor (i.e., write as a product) the following expressions:
(a) $4x^2+8xy+4y^2$
(b) $9x^2-25$
(c) $(x-2)^2-(y+3)^2$
(d) $(x-2)^2-10(x-2)+25$
(e) $256-a^8b^8$
(f) $3x^3-x^2y+6x^2y-2xy^2+3xy^2-y^3$
(g) $a^2-b^2-10b-25$
(h) x^4+4 [Hint: add and then subtract $4x^2$.]

2. Solve the following equations.
(a) $5(x+1)=3x+2$
(b) $(x^2-1)(x+2)=0$
(c) $\frac{x+2}{x+3}=2$
(d) $(x-3)(x+4)=0$
(e) $x^2+4x=0$
(f) $x^3+4x=0$

3. Prove:

(a) $(a+b)^3 = a^3 + 3a^2b + 3ab^2 + b^3$
(b) $(a-b)^3 = a^3 - 3a^2b + 3ab^2 - b^3$
(c) Find $(a+b)^4$, $(a-b)^4$ using the previous results.

4. A $4 \times 4 \times 4$ cubical box has 64 small cubes inside. How many of these touch a side or the bottom of the box?

5. Amanda has an average of 92 on her seven tests. What should she get on her $8th$ test to have an average of 93?