CHAPTER 24

ROTATIONS

24.1 Homework

This is the last homework of our class, let's get it spinning!

1. Give the minimum angle to carry an equilateral triangle onto itself, using a clockwise rotation around the center of the circle passing through its vertices.

2. Same question for a square.

3. Same question for a pentagon.

4. Consider the figure $ABCD$ with $A(6,4)$, $B(1,3)$, $C(2,2)$ and $D(4,2)$. Determine the coordinates of the figure $EFGH$ obtained by rotating $ABCD$ about the origin with 90° clockwise.

5. Recall that isometries are transformations that preserve distances. Justify why rotations are isometries as follows: take two points A and B at different distances from another point O and an angle α – to keep things simple consider α greater than $\widehat{AOB}$. Rotate A about O with α to obtain A' and do the same for B'. How are triangles ΔAOB and $\Delta A'OB'$ and why ? Hint: use SAS (side, angle (which angle?, why?), side). Conclude.

6. Prove the equivalencies

   (a) 90° counterclockwise rotation about the origin $\Longleftrightarrow (x,y) \mapsto (-y,x)$

   (b) 180° counterclockwise rotation about the origin $\Longleftrightarrow (x,y) \mapsto (-x,-y)$

   (c) 90° clockwise rotation about the origin $\Longleftrightarrow (x,y) \mapsto (y,-x)$

   Hint: you can prove that a point A' is the image by some rotation of a point A about some other point O and with an angle α by simply showing that they respect the definition: you show that $OA = OA'$ and that $\widehat{AOA'}$ is the angle α. Here you can go coordinate by coordinate and then conclude with e.g. Pythagoras.