

## 18.1 Homework

1. Factorize the following:

(a)  $a^8 - b^8$

(b)  $a^2 + 2ab + b^2 - c^2$

(c)  $a^2 + b^2 + c^2 + 2(ab + ac + bc)$

2. Find the solutions of the following polynomial equations by factoring:

(a)  $x^3 - 10x^2 + 21x = 0$

(b)  $\frac{x^4 - 8x^2 + 16}{x - 2} = 0$

(c)  $|x^6 - 8x^3 + 16| - 16 = 0$

3. State and prove the Vieta relations for  $n = 3$ , that is, for a polynomial  $P(x) = a_3x^3 + a_2x^2 + a_1x + a_0$  and  $x_1$ ,  $x_2$  and  $x_3$  its roots.

4. Let  $P(x) = x^3 - 3x - 9000$ . Find  $x_1^3 + x_2^3 + x_3^3$  where  $x_1$ ,  $x_2$  and  $x_3$  are the roots of  $P(x)$ .

5. Give the first two terms of the geometric progression

$$a_1, a_2, 12, 16, \dots$$

6. For what positive values of  $x$  and  $y$  would  $\sqrt{xy} = \frac{x+y}{2}$  be true ?

7. Solve the following equations:

(a)  $|x + 2| = 3$

(b)  $|x + 1| - |x - 3| + 2 = 0$

8. Solve the following inequalities. Also show the solution on the real line, and write the answer in the interval notation

(a)  $|x - 3| > 2$

(b)  $|x + 1| - |x - 3| + 2 > 0$

9. Graph the following lines

(a)  $|x + y| = 1$

- (b)  $y = x^2$
- (c)  $y = x^2 + 7$
- (d)  $y = (x + 7)^2$
- (e)  $x^2 + y^2 = 4$
- (f)  $x^2 - 6x + 9 + y^2 + 4y + 4 = 1$

10. Graph the following surfaces

- (a)  $x + y \geq 0$  (hint: graph  $x + y = 0$  and study one point outside it).
- (b)  $x + y \geq 0$  and  $x \in [-1, 1]$  and  $y \in [-1, 1]$
- (c)  $x - y < 0$  and  $x + y > 0$  and  $y < 4$
- (d)  $|x + y| < 1$
- (e)  $x^2 + y^2 < 1$

11. Suppose for each homework problem from your regular math class from school your parents offer you a small box of biscuits. Also, for each homework problem from your School Nova math class, your parents offer you a bag of M&M's. The price of such a biscuit box is \$2 and the price of such an M&M's bag is \$3, but your parents have limited the total monthly budget to \$48 and the total number of items per month to 20.

You are of course tempted to gobble up the most sweets, so you are thus trying to maximize your sugar intake. Such a box of biscuits brings you 10g of sugar and such a bag of M&M's brings you 12g of sugar. What is then the optimal choice for biscuits and M&M's for you in a given month ?

Hint: start by denoting with  $x$  some number of biscuit boxes and by  $y$  some number of M&M's bags and write all these quantities (their combined price, their combined sugar, the number of items) and their limitations. Then think graphically, similar to the previous problems, drawing lines and surfaces, and also formulate geometrically what you are after, so you can find it algebraically and then numerically.