CHAPTER 16

MORE VIETA, FACTORIZATIONS AND THE AM-GM INEQUALITY

Homework

1. Finish all the previous homework problems and try to do as many of these ones.

2. A container C has the shape of a cube with the size of 5m. We would like to build a container D, with the same shape, but with a volume of 9000 cubic meters. By how many meters is the side of D larger than the side of C?

3. For any a and b two positive real numbers we know that $a \geq b$ is equivalent to $a^2 \geq b^2$ (they are either both true or both false). Using this fact, prove the AM-GM inequality for two numbers:

   $$\frac{a+b}{2} \geq \sqrt{ab} \qquad \text{for any positive } a \text{ and } b$$

   Hint: just square it up, and move terms around until you recognize what you have and then conclude using $c^2 \geq 0$ for any real number c.

4. If $x + y = 8$ and they are both strictly positive (so none is zero), what is the maximum value of the product $\sqrt{xy}$? Hint: use the inequality from the previous problem.

5. If $x + y = 8$ and they are both strictly positive, what is the maximum value of the product xy ?

6. If $x + y = 8$ and they are both strictly positive, what is the minimum value of $\frac{1}{xy}$?

7. If $x + y = 8$ and they are both strictly positive, find the minimum value of the expression

   $$\left(1 + \frac{1}{x}\right)\left(1 + \frac{1}{y}\right)$$

   Hint: rewrite by bringing to the same denominator, and use the previous problems.

8. If the perimeter of a rectangle is 17m, what is the greatest possible area ?

9. Prove that $(a + b)\left(\frac{1}{a} + \frac{1}{b}\right) \geq 4$ for a and b strictly positive real numbers. Hint: expand and see if you can use the AM-GM inequality on some combination of a and b where luckily one of AM-GM sides would be easy to compute numerically.