CHAPTER 6

SUPPLEMENTAL ALGEBRA PROBLEMS

1. Prove that there are no counting numbers satisfying $3x^4 + 5y^2 + 5y = 7^5$.
2. Determine a gcd for $A = 1 \times 2 \times 3 \times \cdots \times 100$ and $B = 3^{25} \times 7^6$.
3. Show that for any natural number n we cannot simplify further the fraction $\dfrac{4n+3}{3n+2}$.
4. Find all the natural numbers x, y such that $x + y^3 - y^2 + 2016 = 2017$.
5. In the cross country final there are 1500 high schoolers. Prove that at least two of them know the same number of students among the other competitors (we assume that if A knows B, then B knows A as well).
6. Show that $S = 1 + (3^2 - 2^2) + (3^3 - 2^3) + \cdots + (3^{2011} - 2^{2011})$ is divisible by 5.
7. Let x, y, z be positive rational numbers such that
$$\frac{2x}{3y+4z} = \frac{3y}{2x+4z} = \frac{4z}{2x+3y}.$$
Find a numerical value for
$$(x+3y+2z)\left(\frac{1}{x} + \frac{1}{3y} + \frac{1}{2z}\right)$$
and
$$\frac{x}{y} + \frac{y}{z} + \frac{z}{x}$$