COORDINATE GEOMETRY MATH6

Cartesian coordinate system

Coordinate axes: Two perpendicular lines that intersect at the origin O on each line. We consider the horizontal line with positive direction to the right, and we call it the x-axis. We consider the vertical line with positive direction upward and we call it the y-axis.
Representation of point P in the plane: Draw lines through P perpendicular to the x-and y-axes. These lines intersect the axes in points with coordinates x_0 and y_0. We say that $P(x_0, y_0)$ is the point with x-coordinate x_0, and y-coordinate y_0.
The midpoint of a segment AB with endpoints $A(x_1, y_1)$ and $B(x_2, y_2)$ has coordinates $\left(\frac{x_1 + x_2}{2}, \frac{y_1 + y_2}{2}\right)$.

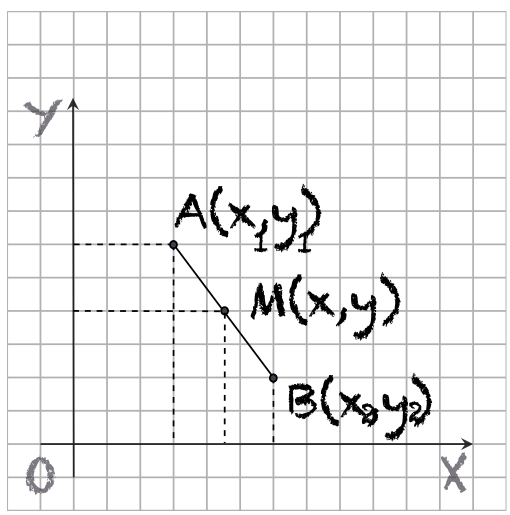


Figure 12.1: Midpoint

Pythagorean Theorem

The square of the hypotenuse of a right triangle is equal to the sum of the squares of the two legs, i.e. $a^2 = b^2 + c^2$.
Some of the Pythagorean Triples are : (3,4,5), (5,12,13), (7,24,25), (8,15,17), (9,40,41), (11,60,61), (12,35,37), (13,84,85), (15,112,113), (16,63,65), (17,144,145), (19,180,181), (20,21,29), (20,99,101)

Distance Formula

The distance between the points $P(x_1, y_1)$ and $Q(x_2, y_2)$ is $d = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$
Proof Use Pythagorean Th

Straight Lines. Parallel and Perpendicular Lines

The slope of a straight non vertical line measures the rate of change of y with respect to x.
The slope of a non vertical line that passes through the $P(x_1, y_1)$ and $Q(x_2, y_2)$ is

$$m = \frac{y_2 - y_1}{x_2 - x_1}$$

The slope of a vertical line is not defined.

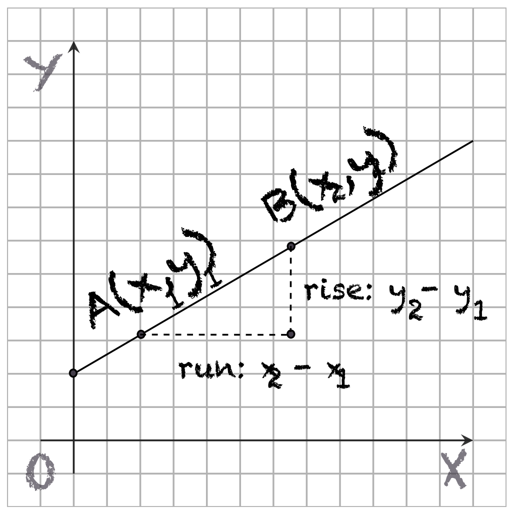


Figure 12.2: Slope: run and rise

y-intercept form: The equation of a straight line $y = mx + b$, m=slope, b= y-intercept (the line passes through $(b, 0)$)
x,y-intercept form: The equation of a straight line whose x and y intercepts are a and b, (the line passes through $(0, a)$ and $(b, 0)$) respectively, is: $\frac{x}{a} + \frac{y}{b} = 1$
Two non vertical lines are parallel if and only if they have the same slope.
Two non vertical lines with slopes m and n are perpendicular if and only if $m \cdot n = -1$

Circle

The equation of the circle with center $O(x_0, y_0)$ and radius r is

$$(x - x_0)^2 + (y - y_0)^2 = r^2$$

Proof Use Pythagoras Th. : for any point $P(x, y)$ on the circle the distance $OP = r$

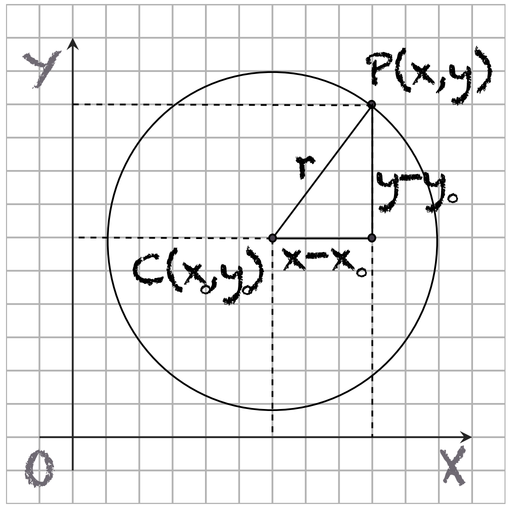


Figure 12.3: Circle equation

Problems

1. A point B is 5 units above and 2 units to the left of point A(7, 5). What are the coordinates of point B?

2. Plot on the coordinate plane the following dots and connect each dot to the next one. If you did everything correctly, you will get a picture. (0,2); (0,0); (1,3); (2,3); (3,2); (3,0); (1,-1); (2,-1); (1,-3); (0,-1); (-1,-3); (- 2,-1); (-1,-1); (-3,0); (-3,2); (-2,3); (-1,3); (0,0)

3. Find the coordinates of the midpoint of the segment AB, where A = (3, 11), B = (7, 5).

4. Draw points A(4,1), B(3,5), C(?1,4). If you did everything correctly, you will get 3 vertices of a square. What are coordinates of the fourth vertex? What is the area of this square?

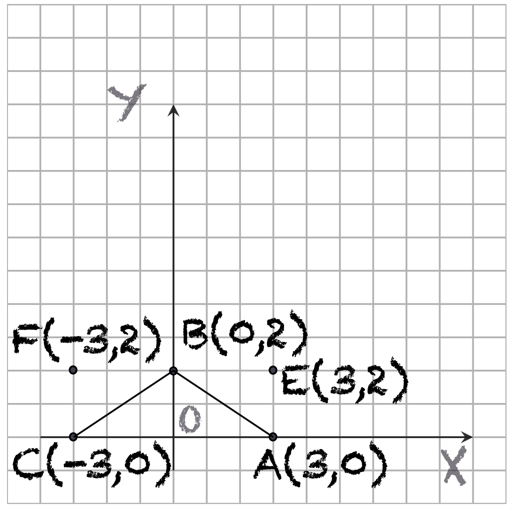
5. (a) 3 points A(0, 0), B(1, 3), D(5, ?2) are vertices of a parallelogram ABCD. What are the coordinates of point C?
   (b) 3 points A(0, 0), B(2, 3), D(4, 1) are vertices of a parallelogram ABCD. What are the coordinates of point C?
   (c) 3 points A(0, 0), B(1, 5), D(3, ?2) are vertices of a parallelogram ABCD. What are the coordinates of point C?
   (d) Can you guess the general rule: if A(0, 0), B(b1, b2), D(d1, d2) are 3 vertices of a parallelogram, what are coordinates of point C?

6. What is the diagonal distance across a square of size 1?

7. The sizes of the sides of a triangle are : $3n, 4n$, and $5n$. What type of triangle is it?

8. Consider the triangle ΔABC with the vertices $A(-2,-1), B(2,0), C(2,1)$. Find the coordinates of the midpoint of B and C. Find the length of the median (i.e. a median unites a vertex with the midpoint of the opposite side) from A in the triangle ΔABC.

9. Consider the triangle ΔABC with the vertices $A(-3,0), B(0,2), C(3,0)$.
   (a) Which type of triangle is the triangle ΔABC. Find its area.



Exercise 8: Triangle, Collinear points

(b) If we add the points $E(3,2)$ and $F(-3,2)$, are the points F, B and E collinear? Why?

(c) Which type of figure is ACFE? Identify the properties that your should have. Can you check them using the slopes?

10. Consider the point $P(4,1)$.

    (a) How many lines passing through P have the slope $\frac{1}{4}$? Find their equations.

    (b) How many lines passing through P have the slope -4? Find their equations.

    (c) Find all the vertical lines in P and write down their equations.

    (d) Find all the horizontal lines in P and write down their equations.

    (e) Find the equations of all the lines in P which intersect the x-axis at a 45 degree angle.

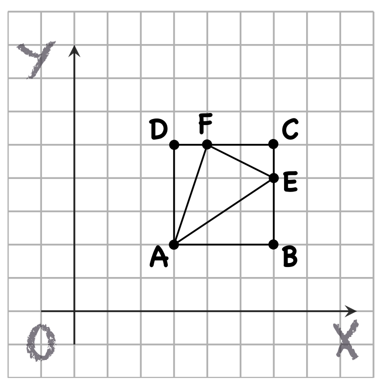
11. Consider the rectangle ABCD and the points $A(3,2), E(6,4)$ and $F(4,5)$.

    (a) Find the coordinates of B, C, and D.

    (b) Find the area of ΔAEF.

12. Consider the circle given by the equation $4x^2 + 4y^2 = 25$ Find its radius and the coordinates of its center.

13. Consider the circle C of equation $(x+1)^2 + (y-2)^2 = 10$. If $A(2,1)$ is the extremity of one diameter of the circle C, find the coordinates of the other extremity of the diameter.



Exercise 11: Rectangle