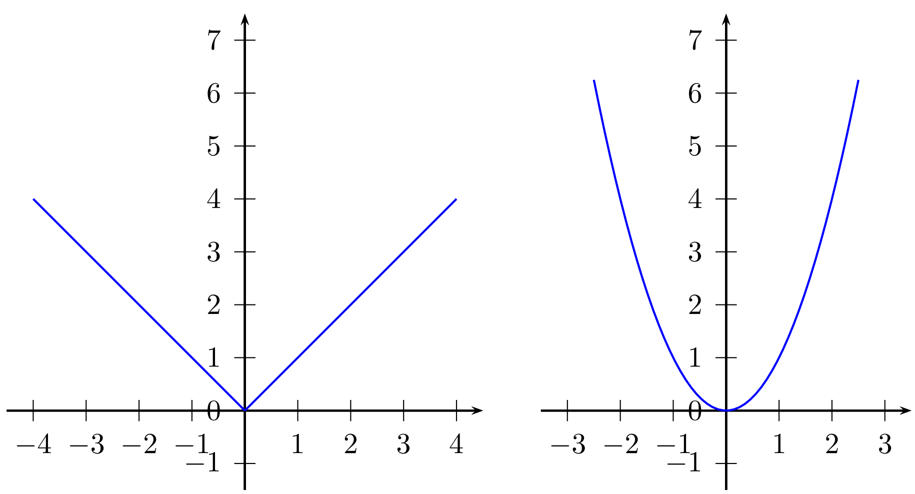
MATH 6: HANDOUT 23
COORDINATES III

GRAPHS

Generally, a graph of function $y = f(x)$ is some line in the $x - y$ plane. If one has two graphs $y = f(x)$ and $y = g(x)$ one can find intersection points of corresponding graphs by solving the *system of equations*. For example, the intersection point of two straight lines $y = x + 2$ and $y = -x$ is the point $(-1, 1)$ as $x = -1$ and $y = 1$ satisfy both of these equations that is the point $(-1, 1)$ lies simultaneously on both straight lines.

GRAPHS OF $y = |x|$ AND $y = x^2$

The figures below show graphs of functions $y = |x|$ and $y = x^2$; the latter graph is called a *parabola*.



And here is what we can do to draw a graph of any parabola of the sort $y = ax^2 + bx + c$. You can verify the following identity yourself:

$$ax^2 + bx + c = a\left(x + \frac{b}{2a}\right)^2 - \frac{b^2 - 4ac}{4a} = a(x-h)^2 + k, \qquad h = \frac{b}{2a}, \qquad k = -\frac{b^2 - 4ac}{4a}.$$

For example: $x^2 + x = (x + \frac{1}{2})^2 - \frac{1}{4}$

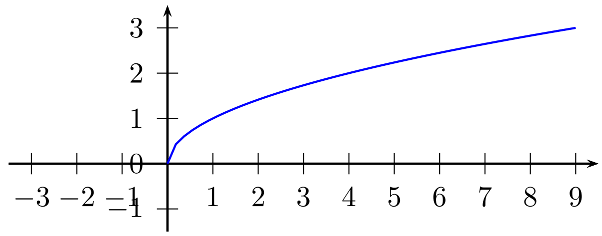
The result will be a parabola obtained by stretching the usual parabola vertically by factor a (if $a < 0$, this means flipping it upside down and then stretching by $|a|$) and then moving it so that the vertex will be at point (h, k).

In particular, the branches go up if $a > 0$ and down if $a < 0$.

Obviously the parabola either intersects $y = 0$ at two points or does not intersect it or touches $y = 0$ at a single point. Correspondingly the quadratic equation has two roots, no roots or one root respectively. One can easily check that this corresponds to $D > 0$, $D < 0$ and $D = 0$ respectively, where $D = b^2 - 4ac$.

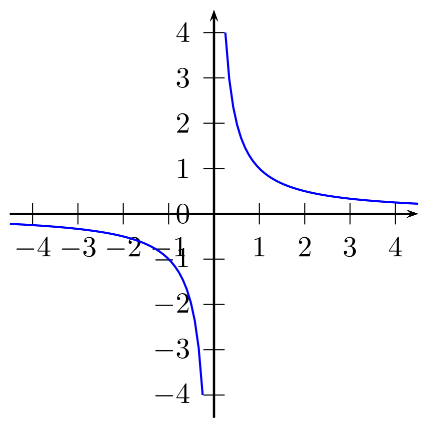
Graph of $y = \sqrt{x}$

This graph is similar to the graph of $y = x^2$ but it "lies on its side". Notice, that it is not defined for $x < 0$.



Graph of $y = \frac{1}{x}$

The graph of the inverse function $y = \frac{1}{x}$ is different from other graphs in that it is *discontinuous*: it is not defined when $x = 0$.



Homework

1. Find the equation of the line through $(1, 2)$ with slope -2.

2. Find the equation of the line through points $(-1, 2)$ and $(2, 1)$.

3. Find the intersection point of a line $y = x - 3$ and a line $y = -2x + 6$. Sketch the graphs of these lines.

4. Sketch graphs of the following functions:

(a) $x + y = 2$ (b) $y = |x - 5| + 1$ (c) $y = |x + 1| + |x - 2|$

(d) $y = (x - 1)^2 + 1$ (e) $y = -x^2 + 4x - 3$ (f) $|x + y| = 2$

(g) $y = |x^2 - x|$ (h) $y = \sqrt{x - 3}$ (i) $y = \dfrac{1}{x + 2} + 1$

(j) $y = \dfrac{1}{2 - x}$ (h) $\dfrac{1}{x - 1} + 1$ (i) $y = \dfrac{x + 2}{x + 1}$