MATH 6: HANDOUT 17
INVARIANTS AND SEMI-INVARIANTS

Homework

Invariant: something that remains unchanged
Semi-invariant: something that changes in one direction only (for example, a number that always increases).

1. (a) We are given a 4×4 table, each cell containing either $+$ sign or $-$ sign:

    ```
    +  −  +  +
    +  +  +  +
    +  +  +  +
    +  −  −  +
    ```

    You can reverse all signs in a single row or column, replacing each $+$ by $-$ and $-$ by $+$. Is it possible to make all signs $+$ by repeating this operation?

    *(b) Same question, but for this table:

    ```
    +  −  +  +
    +  +  +  +
    +  +  +  +
    +  −  +  +
    ```

2. Andrew is angry with his grades so he is tearing his report card into pieces: every time he takes one of he pieces and tears it into 5. Can he get a) 26 b) 2011 pieces?

3. Now Nick is helping Andrew to tear his report card, so they are tearing it together. However, Nick is tearing every piece into 7 ones. Can they get a) 2010 b) 2011 pieces?

4. A hundred numbers, 1 through 100, are written on the blackboard. Every minute one of this numbers is increased by 1, and another is decreased by 2. Show that after some time the sum will become negative.

5. 100 different numbers are written in a row on the board. Every second you can do the following operation: if two numbers next to each other are in a wrong order (i.e., the larger number is before the smaller number), you can exchange them. Show that after some time all numbers will be in the correct order.

6. Each of n countries is ruled by one of two parties (let us call them "red" and "blue"). If a country A is ruled by a red party, but majority (more than a half) of its neighbors are ruled by blue party, then the blue party can use their support to overthrow the government of the country A, so that A also becomes ruled by blue party. Of course, the same applies to the red party. Show that it can not continue indefinitely: after some time, no more regime changes will be possible.