MATH 6
HANDOUT 7: SETS CONTINUED

Counting

We denote by $|A|$ the number of elements in a set A (if this set is finite). For example, if $A = \{a, b, c, \dots, z\}$ is the set of all letters of English alphabet, then $|A| = 26$.

If we have two sets that do not intersect, then $|A \cup B| = |A| + |B|$: if there are 13 girls and 15 boys in the class, then the total is 28.

If the sets do intersect, the rule is more complicated:

$$|A \cup B| = |A| + |B| - |A \cap B|$$

(see problem 6 below).

Product rule

If we need to choose a pair of values, and there are a ways to choose the first value and b ways to choose the second, then there are ab ways to choose the pair.

For example, a position on a chessboard is described by a pair like a4; there are 8 possible choices for the letter, and 8 possible choices for the digit, so there are $8 \times 8 = 64$ possible positions.

It works similar for triples, quadruples, ... For example, if we toss a coin, there are 2 possible outcomes, heads (H) or tails (T). If we toss a coin 4 times, the result can be written by a sequence of four letters, e.g. HTHH; since there are 2 possibilities for each of the letters, we get $2 \times 2 \times 2 \times 2 = 2^4 = 16$ possible sequences we can get.

1. Let $A = [1, 3] = \{x \mid 1 \le x \le 3\}$, $B = \{x \mid x \ge 2\}$, $C = \{x \mid x \le 1.5\}$. Draw on the number line the following sets: $\overline{A}$, $\overline{B}$, $\overline{C}$, $A \cap B$, $A \cap C$, $A \cap (B \cup C)$, $A \cap B \cap C$.

2. Long ago, in some town a phone number consisted of a letter followed by 3 digits (e.g. K651). How many possible phone numbers could there be in that town?[Note: digits could be zero, so a number like X000 was allowed.]

3. If we roll 3 dice (one red, the other white, and the third one, black), how many combinations are possible? How many combinations in which the sum of values is exactly 4?

4. A **subset** of a set A is a set formed by taking some (possibly all) elements of A; for example, the set $\{2, 4, 6, 8\}$ is a subset of the set $\{1, 2, 3, 4, 5, 6, 7, 8, 9\}$.

List all subsets of the set $S = \{1, 2, 3\}$ (do not forget the empty set which contains no elements at all and S itself).

Can you guess the general rule: if set S has n elements, how many subsets does it have?

5. (a) Using Venn diagrams, explain why $\overline{A \cap B} = \overline{A} \cup \overline{B}$. Does it remind you of one of the logic laws we had discussed before?

(b) Do the same for formula $A \cap (B \cup C) = (A \cap B) \cup (A \cap C)$.

6. In this problem, we denote by $|A|$ the number of elements in a finite set A.

(a) Show that for two sets A, B, we have $|A \cup B| = |A| + |B| - |A \cap B|$.

*(b) Can you come up with a similar rule for three sets? That is, write a formula for $|A \cup B \cup C|$ which uses $|A|$, $|B|$, $|C|$, $|A \cap B|$, $|A \cap C|$, $|B \cap C|$.

7. In a class of 33 students, 12 are girls, 10 play soccer, and 10 play chess. Moreover, it is known that 6 of the soccer players are girls, that 2 of the chess players also play soccer, and that there is exactly one girl who plays both chess and soccer. Finally, 4 girls play neither soccer nor chess. Can you figure out how many boys play soccer? chess? both? neither?

8. 150 people at a Van Halen concert were asked if they knew how to play piano, drums or guitar.
(a) 18 people could play none of these instruments.
(b) 10 people could play all three of these instruments.
(c) 77 people could play drums or guitar but could not play piano.
(d) 73 people could play guitar.
(e) 49 people could play at least two of these instruments.
(f) 13 people could play piano and guitar but could not play drums.
(g) 21 people could play piano and drums.
How many people can play piano? drums?

***9.** A barber in a small town decides that he will shave all men who do not shave themselves (and only them). Should he shave himself? [Of course, the barber is a man.]