

MATH 6

HANDOUT 3: (KNIGHTS AND KNAVES) OR (LADIES AND TIGERS)

Today we discussed the meaning of words “and”, “or”, “if”

AND (for example $A \text{ AND } B$): true if both A, B are true, and false otherwise

OR (for example $A \text{ OR } B$): true if at least one of A, B is true, and false otherwise. Sometimes also called “inclusive or”

IF: when someone says “if A then B ”, and A is false, do you think he lied? for example, is the statement

“if sky is green, then $2+2=5$ ”

true?

The usual convention in mathematics is that it is true: any statement beginning with the words “if A then.. ” is taken to be true in the case when A is false.

1. Solve the following equations:

$$(a) 2x - 22 = 3(1 - x) \quad (b) 1 - \frac{2}{7}x = \frac{1}{7}x \quad (c) 1 - 8(1 - x) = 7x - 8$$

2. On the island of Knights and Knaves, you meet two inhabitants: Carl and Bill. Carl says, “I and Bill are both knights or both knaves.” Bill claims, “Only a knave would say that Carl is a knave.” [Hint: first, rewrite Bill’s claim in an easier to understand form.]
3. On the island of Knights and Knaves, you meet three inhabitants: Bob, Mel and Peggy. Bob says that it’s not true that Peggy is a knave. Mel says that Peggy is a knight or Bob is a knave. Peggy claims, “Both I am a knight and Bob is a knave.”
4. On the island of Knights and Knaves, you meet three inhabitants: Bozo, Carl and Joe. Bozo says that Carl is a knave. Carl tells you, ‘Of Joe and I, exactly one is a knight.’ Joe claims, ‘Bozo and I are different.’
5. Once upon a time there was a king who had many beautiful daughters. There were many dangerous tigers in his kingdom as well. A knight came to the king. He wanted to marry a princess. The king decided to test how clever the knight was. He offered the following test. There were three doors leading to three rooms. In one of the rooms, there was a princess waiting for the knight. In another, there was a hungry tiger. The third room was empty.

The king also placed the signs on the doors — but the sign on the door of princess’ room was true, the sign on the tiger’s door was false, and the sign on the door of the empty room could be either false or true.

Here are the signs:

Room I	Room II	Room III
Room III is empty	The tiger is in room I	This room is empty

Which door should the knight open?

6. A certain convention numbered 100 politicians. Each politician was either crooked or honest. We are given the following two facts:
 - At least one of the politicians was honest.
 - Given any two of the politicians, at least one of the two was crooked.

Can it be determined from these two facts how many of the politicians were honest and how many of them were crooked?

7. An inhabitant, Carl, of the island Knights and Knaves tells you “If Sue is a knave, then this road leads to the capital.” However, later you learn that Carl is a knave. What does it tell you?
8. Many trucks carry the message: “If you do not see my mirrors, then I do not see you”. Can you rewrite it in an equivalent form without using the word “not”?
9. On the island of Knights and Knaves, a traveler meets two inhabitants: Carl and Bill. Bill says: “Carl is a Knave”. Carl says: “If Bill is a Knight, then I am a Knight, too.”
Can you determine whether each of them is a Knight or a Knave?
10. A mom tells the son “If you do not do the dishes, you will not go to the movie”. Is it the same as “If you do the dishes, you go to the movie?”
- *11. (The starred problems are those which are more difficult and thus are optional.)
On the island of Knights and Knaves, you come to a fork in the road. You know that one of the roads leads to the capital, but you do not know which one. However, there is a man at the crossroads. Can you find out which road leads to the capital by asking him one question that can be answered “yes” or “no”?
- *12. And now, for something completely different...
Three gods A, B, and C are called, in some order, True, False, and Random. True always speaks truly, False always speaks falsely, but whether Random speaks truly or falsely is a completely random matter. Your task is to determine the identities of A, B, and C by asking three yes-no questions; each question must be put to exactly one god.