

MATH 6
HANDOUT 2: LOGIC I. KNIGHTS AND KNAVES

Many of the questions of this assignment refer to the famous (among logic puzzle fans) island of Knights and Knaves. On this island, there are two kinds of people: Knights, who always tell the truth, and Knaves, who always lie. Unfortunately, there is no easy way of knowing whether a person you meet is a knight or a knave. . .

Copyright notice: most of these problems come from books of Raymond Smullyan. If you liked them, get his books in the library and you will find there many more puzzles of the same sort. You can also find a number of such puzzles online at <http://philosophy.hku.hk/think/logic/puzzles.php>

1. You meet two inhabitants: Peggy and Zippy. Peggy tells you that ‘of Zippy and I, exactly one is a knight’. Zippy tells you that only a knave would say that Peggy is a knave.
Can you determine who is a knight and who is a knave?
2. You meet two inhabitants: Sally and Zippy. Sally claims, ‘I and Zippy are not the same.’ Zippy says, ‘Of I and Sally, exactly one is a knight.’
Can you determine who is a knight and who is a knave?
3. You meet two inhabitants: Marge and Zoey. Marge says, ‘Zoey and I are both knights or both knaves.’ Zoey claims, ‘Marge and I are the same.’
Can you determine who is a knight and who is a knave?
4. You meet two inhabitants: Mel and Ted. Mel tells you, ‘Either Ted is a knight or I am a knight.’ Ted tells you that Mel is a knave.
Can you determine who is a knight and who is a knave?
5. You meet two inhabitants: Ted and Zeke. Ted claims, ‘Zeke could say that I am a knave.’ Zeke claims that it’s not the case that Ted is a knave.
6. You meet two inhabitants: Ted and Zippy. Ted says, ‘Of I and Zippy, exactly one is a knight.’ Zippy says that Ted is a knave.
Can you determine who is a knight and who is a knave?
7. You meet two inhabitants: Bob and Betty. Bob claims that Betty is a knave. Betty tells you, ‘I am a knight or Bob is a knight.’
Can you determine who is a knight and who is a knave?
8. You meet two inhabitants: Bart and Mel. Bart claims, ‘Both I am a knight and Mel is a knave.’ Mel tells you, ‘I would tell you that Bart is a knight.’
Can you determine who is a knight and who is a knave?
9. You meet two inhabitants: Alice and Ted. Alice tells you, ‘Either Ted is a knave or I am a knight.’ Ted tells you, ‘Of I and Alice, exactly one is a knight.’
Can you determine who is a knight and who is a knave?
10. You meet two inhabitants: Ned and Zoey. Zed says that it’s false that Zoey is a knave. Zoey claims, ‘I and Ned are different.’
Can you determine who is a knight and who is a knave?
11. You meet two inhabitants: Sue and Marge. Sue says that Marge is a knave. Marge claims, ‘Sue and I are not the same.’
Can you determine who is a knight and who is a knave?
12. You meet two inhabitants: Carl and Betty. Carl says, ‘Neither Betty nor I are knaves.’ Betty claims, ‘Carl and I are the same.’
Can you determine who is a knight and who is a knave?
13. Alice, Brian, and Charlie are from the island of knights and knaves. Alice claims, “Charlie could tell you that I am a knight.” Brian says, “Either Alice is a knave, or I am a knight.”

Charlie says that the others are either both knaves or both knights. What are Alice, Brian, and Charlie?

14. Now imagine that the island also have normals, who can either say truth or lie. Amy, Bob, and Celine are from the island of knights, knaves, and normals. One of them is a knight, one is a knave, and one is normal. Amy says that Celine is a knave. Bob says that Amy is a knight. Celine says that she is a normal. Can you figure out who is who?

HOMEWORK

1. Find the greatest common divisor and least common multiple of 132 and 90.
2. Solve the following equation: $5 - 2(1 - x) = 9$.
3. On the island of knights and knaves, you meet two inhabitants: Zoey and Mel. Zoey tells you that Mel is a knave. Mel says, "Neither Zoey nor I are knaves." So who is a knight and who is a knave?
4. On the island of knights and knaves, you meet two inhabitants: Sue and Zippy. Sue says that Zippy is a knave. Zippy says, "I and Sue are knights." So who is a knight and who is a knave?
5. On the island of knights and knaves, you meet two inhabitants: Bart and Ted. Bart claims, "I and Ted are both knights or both knaves." Ted tells you, "Bart would tell you that I am a knave." So who is a knight and who is a knave?
6. You meet two inhabitants: Betty and Peggy. Betty tells you that Peggy is a knave. Peggy tells you, 'Betty and I are both knights.'
Can you determine who is a knight and who is a knave?
7. You meet two inhabitants: Zed and Peggy. Zed says that Peggy is a knave. Peggy tells you, 'Either Zed is a knight or I am a knight.'
Can you determine who is a knight and who is a knave?
8. You meet two inhabitants: Zed and Alice. Zed tells you, 'Alice could say that I am a knight.'
Alice claims, 'It's not the case that Zed is a knave.'
Can you determine who is a knight and who is a knave?
9. A traveler to the island of Knights and Knaves meets a group of five people (call them A, B, C, D, E).
A says: "exactly one of us is a Knight"
B says: "exactly two of us are Knights"
C says: "exactly three of us are Knights"
D says: "exactly four of us are Knights"
E says: "all five of us are Knights"
Can you find out which of them are Knights?
10. You are in a maze on the island of knights and knaves. There are two doors: you know that one leads to freedom and one leads to certain doom. There are two guards nearby, and you happen to know that one is a knight and one is a knave, but you don't know who is who. They allow you to ask one of them a single question before you choose a door — what do you ask?