

MATH 6: HOMEWORK 10

ARITHMETIC SEQUENCES

1. ARITHMETIC SEQUENCE

A sequence of numbers (typically but not always integers) is an arithmetic sequence if the difference between consecutive terms is the same number, the **common difference**, let's call it d .

For example, let's consider the sequence: 1, 5, 9, 13, 17, ...

The first term in the sequence is $a_1 = 1$, the second is $a_2 = 5$, and so on. The difference is $d = 4$.

What is the n^{th} term? For example what is a_{100} ?

$$a_1 = 1$$

$$a_2 = a_1 + d = 1 + 4 = 5$$

$$a_3 = a_2 + d = (a_1 + d) + d = a_1 + 2d = (1 + 4) + 4 = 1 + 2 \times 4 = 9$$

$$a_4 = a_3 + d = (a_2 + d) + d = ((a_1 + d) + d) + d = a_1 + 3d = 1 + 3 \times 4 = 13$$

....

$$a_n = a_1 + (n - 1)d$$

$$\text{So } a_{100} = a_1 + 99d = 1 + 99 \times 4 = 397$$

2. PROPERTY OF AN ARITHMETIC SEQUENCE

A property of an arithmetic sequence is that any term is the arithmetic mean of its neighbors.

$$a_n = \frac{a_{n-1} + a_{n+1}}{2}$$

To prove this we can write:

$$\begin{aligned} a_n &= a_{n-1} + d \\ a_n &= a_{n+1} - d \end{aligned}$$

Add and we have $2a_n = a_{n-1} + a_{n+1}$, and $a_n = \frac{a_{n-1} + a_{n+1}}{2}$

To find the common difference between 2 terms a_s and a_t

$$d = \frac{a_s - a_t}{s - t}.$$

3. SUM OF AN ARITHMETIC SEQUENCE

$$S = a_1 + a_2 + a_3 + \dots + a_n = n \cdot \frac{a_1 + a_n}{2}$$

To prove this, we write the sum in 2 ways, in increasing and decreasing order:

$$S = a_1 + a_2 + a_3 + \dots + a_n$$

$$S = a_n + a_{n-1} + a_{n-2} + \dots + a_1$$

Adding, we notice that $a_1 + a_n = a_2 + a_{n-1} = a_3 + a_{n-2} = \dots$

$$2S = (a_1 + a_n)n \text{ and dividing by } 2 \text{ gives us } S = n \cdot \frac{a_1 + a_n}{2}$$

4. HOMEWORK PROBLEMS

1. $1 + 2 + 3 + \dots + 100 =$
2. $1 + 3 + 5 + \dots + 99 =$
3. $11 + 12 + 13 + \dots + 101 =$
4. Write the first 5 terms of an arithmetic sequence if $a_1 = 7$ and $d = 2$.
5. What are the first 2 terms of the arithmetic sequence $a_1, a_2, -9, -2, 5, \dots$?
6. What is a_1 and what is d for the following arithmetic sequence:
 $-10, -5, 0, 5, 10, \dots$? What is the 25^{th} term?
7. $a_{10} = 131$ and $d = 12$. What is a_1 ?
8. $a_5 = 27$ and $a_{27} = 60$. Find the first term and the common difference.
9. Find the sum of the first 100 terms if $a_1 = -1$ and $d = 1$.
10. Given some integer r and arithmetic sequence $a_1, a_2, a_3, \dots$, let $b_1 = r \cdot a_1$, $b_2 = r \cdot a_2$, $b_3 = r \cdot a_3, \dots$. Prove that $b_1, b_2, b_3, \dots$ is an arithmetic sequence.
11. Prove that, given any arithmetic sequence, if I multiply each term by the same number and then add the same number to each term, the result is still an arithmetic sequence.
12. Simplify the following expression:

$$\frac{\frac{2}{\frac{1}{1 - \frac{1}{3}} - 1}}{\frac{\frac{1}{2}}{\frac{2}{3} - \frac{1}{4}}}$$