

MATH 6: FINAL REVIEW

Recall the subjects that we covered this year:

- Mathematical Logic
- Mathematical Sets
- Combinations, Permutations, and Probability
- Arithmetic Sequences and Geometric Sequences
- Inequalities
- Invariants
- Euclidean Geometry

All the homework sheets for the year are on the webpage for this class on the SchoolNova website, in case you need to or want to print out more copies of past sheets. I will not be copying all the material here into this one sheet (obviously), and **I expect you to review the past sheets**, so you are familiar with the material. **You should be able to describe one or two things you learned from each subject on the list above**, so make sure you are familiar with the general concepts. You do not need to memorize every detail.

There will be an end-of-year test during the upcoming week, and it will cover material from each of the subjects on the above list. The difficulty of the problems will be about the same as the problems on this sheet and on last week's sheet (not including problems 6 and 7 on last week's sheet).

1. HOMEWORK

1. Write out the truth table for each of the following:

$$A \text{ OR } B$$

$$A \implies B$$

$$A \iff B$$

2. Simplify the following logical statement:

$$(A \text{ OR } B) \text{ AND } (A \text{ OR } (\text{NOT } B))$$

3. Let A and B be sets defined as follows: $A = \{0, 1, 2, 3, 4\}$ and $B = \{0, 2, 4, 6, 7\}$.

Describe the sets $A \cup B$ and $A \cap B$.

Is A a subset of B ? Explain why or why not.

4. I have a cake that has several candles and other decorative objects on it. There are 6 candles on the cake, and there are 5 blue objects on the cake. If I know that 9 objects on the cake are blue or a candle, then how many blue candles are there on the cake?
5. Seven runners run a race. The top three to finish are recorded.

How many different possible outcomes are there for the winners of the race? (For example, perhaps person A gets first place and B gets second place and C gets third place; or, instead, maybe B gets first place, D second, and E third, etc.)

The four runners who don't finish in the top three will get a participation medal. How many different possibilities are there for the group of people that gets participation medals?

6. One of the numbers of the following set is chosen at random: $\{1, 2, 4, 8\}$. Call the selected number s . Then, the four Aces and the four Queens of a deck of cards are laid out at random in a s -by- $\frac{8}{s}$ rectangle. What is the probability that the Ace of Hearts is in the top row?
7. Calculate the following sum of numbers: $1 + 2 + 3 + 4 + \dots + 25$.
8. A geometric sequence has its first term $b_1 = 5$ and its fifth term $b_5 = 20$. What is the common ratio of this sequence?
9. Plot the following inequality on a number line: $\frac{x}{x+1} > 0$.
10. Solve the following inequality: $\frac{4}{x} > 4$.

11. A building contains classrooms along two hallways; each hallway has ten rooms on it. Currently there are three students in each of the rooms along the first hallway, and five students in each of the rooms along the second hallway.

How many total students are there in the building?

You have been given permission to swap kids between rooms. You may swap as many times as you want, but each time you must make sure that you pick one student from each of two rooms and that the two students safely switch places. Is it possible for you to swap students around so that the hallways are balanced (i.e., each hallway has the same number of students in it)?

12. Let l be a line in the plane, and let P and Q be two points on opposite sides of l . Let m be the perpendicular from P to l , and let m intersect l at point X .

Using a straightedge and compass, construct a point Y such that $\triangle PXQ \cong \triangle QYP$.

Using a straightedge and compass, construct a point Z on m such that PZQ is a right triangle. Then, construct a point A such that $\triangle PAQ \cong \triangle QZP$.